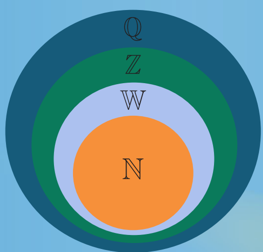
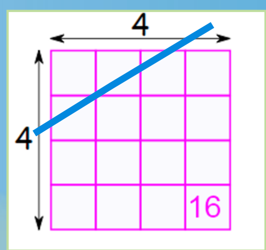
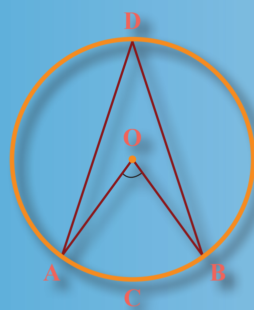
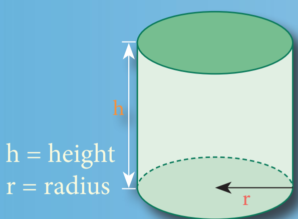
MATHEMATICS

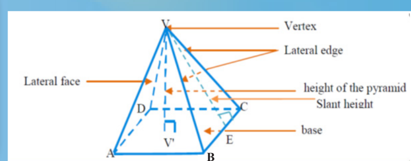
STUDENT'S BOOK

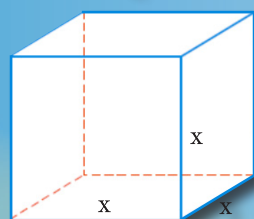


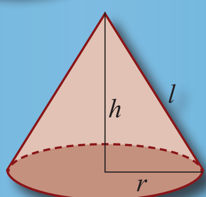


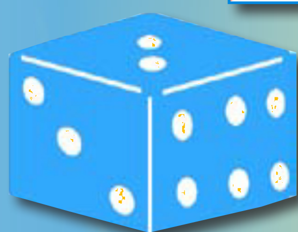












GRADE 8

SNNPR Education Bureau

MATHEMATICS

STUDENT'S BOOK

GRADE 8

Developed by Addis Ababa Educational Bureau and Adapted by South Nations Nationalities and People Regional Education Bureau.

Adapted By:

- Nathan Labiso Langano (MSc)

Layout and Design:

- Sisie Tagesse Bussoro

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Unit 1

1. RATIONAL NUMBERS

Learning Outcomes:

At the end of this unit, learners will able to:

- Define and represent rational numbers as fractions
- Show the relationship among $\mathbb{N}$, $\mathbb{W}$, $\mathbb{Z}$ and $\mathbb{Q}$.
- Order rational numbers.
- Solve problems involving addition, Subtraction, Multiplication and division of rational numbers
- Apply Rational Numbers to solve practical problems.
- Aware the four operations as they relate to Rational Numbers.

Main content

1.1 The Concept of Rational Numbers
1.2 Comparing and Ordering Rational Numbers
1.3 Operation and Properties of Rational Numbers
1.4 Applications of Rational Numbers

- Key terms
- Summary
- Review Exercise

INTRODUCTION

Many times throughout your mathematics lessons, you will be manipulating specific kinds of numbers that are related to your real life activities. So, it is important to understand how mathematicians classify numbers and what kinds of major classifications exist.

In the previous grades you have learnt about the set of Natural numbers, Whole numbers and Integers and their basic properties. In this unit you will learn about the set of numbers which contains the other set of numbers (i,e $\mathbb{N}$, $\mathbb{W}$, $\mathbb{Z}$) it is called Rational numbers. And also you will learn about the basic properties, operations and real life applications of rational numbers.

1.1. The concept of Rational numbers

Competencies:

At the end of this sub-unit, students should:

- Describe the concept of Rational Numbers practically.
- Express Rational Numbers as fractions.

Group Work 1.1

Discuss the following questions with your groups

1. Provide an example of each of the following numbers
   a. Natural numbers smaller than 10.
   b. Whole number that is not a natural number
   c. Integers that is not a whole number

2. Which of the following set of numbers include the other set of numbers?
    a. Whole numbers
    b. Integers
    c. Natural numbers
3. Define a rational number in your own words.
4. Solomon has 3 cats and 2 dogs. He wants to buy a toy for each of his pets. Solomon has 22 Birr to spend on pet toys. How much can he spend on each pet? Write your answer as a fraction and as an amount in Birr and Cents.

1.1.1. Representation of Rational Numbers on a Number line

Competency: **At the end this sub-topic, students should:**

- Represent rational numbers as a set of fractions on a number line.

Revision on Fractions

A fraction represents the portion or part of the whole thing. For example, one-half, three-quarters. A fraction has two parts, namely numerator (the number on the top) and denominator (the number on the bottom).

Example 1.1:

Figure1.1: DVD

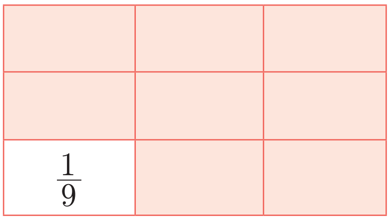


Figure1.2: Rectangular fields

The shaded part is $\frac{7}{8}$ of the DVD

One part is one-ninth, of the rectangular field

Example 1.2:

If three-fifths of the green area be covered by indigenous plants, then find the numerator and denominator of the covered area.

Figure1.3:

Solution: $\frac{3}{5}$ ← Numerator (3), ← Denominator (5)

In grade 6 and 7 you have discussed the important ideas about fractions and integers.

i. Proper fraction: A fraction in which the numerator is less than the denominator.

ii. Improper fraction: A fraction in which the numerator is greater than or equal to the denominator.

If an improper fraction is expressed as a whole number and proper fraction, then it is called mixed fraction.

Integers are represented on number line as shown below in figure 1.4.

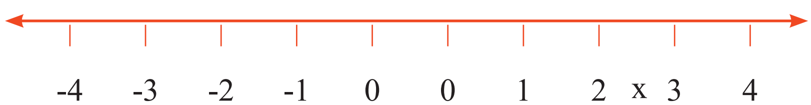


Figure1.4:

What number is represented by the marked letter x on the number line above? You observe that the number x is greater than 2 but less than 3. So, it belongs to the interval between 2 and 3. Thus x is not a natural number, or a whole number, or an integer. What type of a number is?

Using the above discussion, we define a rational number as follow:

Definition 1.1

A number that can be written in the form of $\frac{a}{b}$ where a and b are integers and b $\neq$ 0, is called a rational number.

Example 1.3:

$\frac{1}{5}, \frac{3}{7}, \frac{3}{5}, \frac{-8}{3}, \frac{-14}{4}$ *and* $\frac{5}{9}$ are rational numbers.

Note:	The set of rational numbers is denoted by $\mathbb{Q}$.

How can we locate rational numbers on number line?

Rational number can be represented on number line by following the following steps.

i. Positive rational numbers are always represented on the right side of zero on the number line. While negative rational numbers are always represented on the left side of zero on number line.
ii. Proper fractions always exist between zero and one on number line.
iii. Improper fractions are represented on number line by first converting into mixed fraction and then represented on the number line.

Example 1.4:

Sketch a number line and mark the location of each rational numbers.

a. $\frac{2}{5}$ b. $\frac{3}{2}$ c. $\frac{-3}{4}$ d. $\frac{-5}{2}$

Solution:

a. Since $\frac{2}{5} > 0$, and proper, so it lies on the right side of 0 and on the left side of 1. How can we locate?

Divide the number line between 0 and 1 into 5 equal parts. Then the second part of the fifth parts will be a representation of   on number line.

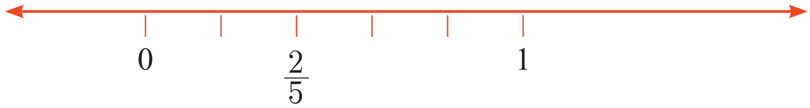


Figure1.5:

b. Since $\frac{3}{2}$ is an improper fraction, first convert to mixed fraction to find between which whole numbers the fraction exists on the number line.

Thus, $\frac{3}{2} = 1\frac{1}{2}$. The fraction would be between 1 and 2 at point. Now, divide the number line between 1 and 2 in two equal parts and then the 1st part of 2 parts will be the required rational number on the number line.

Figure1.6:

c. Since $-1 < \frac{-3}{4} < 0$, the fraction will lie between -1 and 0.To represent on the number line, divide the number line between -1 and 0 in to 4 equal parts and the third part of the four parts will be .

Figure1.7:

d. Since improper, first change in to mixed fraction. That is, . To represent on the number line divide the number line between 3 and 2 in to two equal parts and the first part of the two parts is

Note:	Two rational numbers are said to be opposite, if they have the same distance from 0 but in different sides of 0.

For instance $\frac{3}{2}$ and $-\frac{3}{2}$ are opposites.

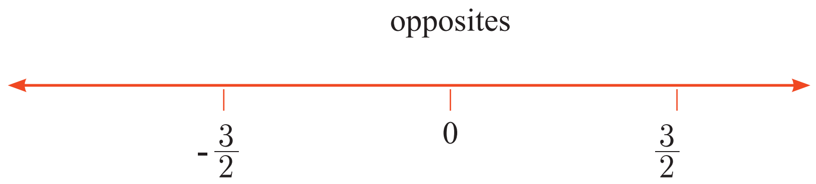


Figure1.8:

Exercise 1.1.

1. Consider the following number line. Select a reasonable value for point B.

Figure1.9:

   a. 0.5 b. 3.6 c. -0.2 d. 2

2. Between what consecutive integers the following rational numbers exist?

   a. $\frac{3}{7}$ b) $\frac{8}{5}$ c) $\frac{-3}{5}$ d) $\frac{-9}{5}$

3. Change the following improper fractions to mixed fractions.

   a. $\frac{32}{5}$ b) $\frac{-27}{10}$ c) $\frac{7}{3}$

4. Represent the following rational numbers on a number line.

   a. $\frac{5}{6}$ b) $\frac{3}{5}$ c) $\frac{-5}{6}$ d) $\frac{-8}{5}$ e) $2\frac{2}{5}$

5. If you plot the point 8.85 on a number line, would you place it to the left or right of 8.8? Explain.

6. Find the opposite of the following rational numbers.

   a. $\frac{4}{5}$ b) $\frac{-2}{3}$ c) $2\frac{3}{5}$ d) $-3\frac{4}{7}$

1.1.2. Relationship among $\mathbb{W}, \mathbb{Z}$ *and* $\mathbb{Q}$

Competency: **At the end of this section, students should:**

- Describe the relationship among the sets , W, and .

In the previous grades of mathematics lesson you have learnt about the sets of natural numbers ($\mathbb{N}$), whole numbers (W) and integers ($\mathbb{Z}$).In this subsection you will discuss the relationship among these set of numbers with the other set of number which is rational numbers.

A collection of items is called a set. A Venn diagram uses intersecting circles to show relationships among sets of numbers. The Venn diagram below shows how the set of natural numbers, whole numbers, integers, and rational numbers are related to each other.

When a set is contained within a larger set in a Venn diagram, the numbers in the smaller set are members of the larger set. When we classify a number, we can use Venn diagram to help figure out which other sets, if any, it belongs to.

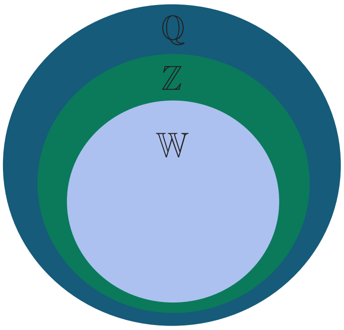


Figure1.10: Venn diagram

Example 1.5

Classify the following numbers by naming the set or sets to which it belongs.

a. -13 b. $\frac{1}{7}$ c. $\frac{-5}{76}$ d. 10

Solution:

a. integer, rational number

b. rational number

c. rational number

d. natural number, whole number, integer, rational number.

Example 1.6:

Is it possible for a number to be a rational number that is not an integer but is a whole number? Explain.

Solution: No, because a whole number is an integer.

Exercise 1.2.

1. Solomon says the number 0 belongs only to the set of rational numbers. Explain his error.
2. Write true if the statement is correct and false if it is not.

   a) The set of numbers consisting of whole numbers and its opposites is called integers.

   b) Every natural number is a whole number.

   c) The number $-3\frac{2}{7}$ belongs to negative integers.

1.1.3. Absolute value of Rational numbers

Competency: **At the end of this section, students should**

☞ Determine the absolute value of a rational number.

Activity: 1.1.

1. What is the distance between 0 and 5 on the number line? and between 0 and -5 on the number line?

The absolute value of a rational number describes the distance from zero that a number is on a number line without considering direction.

For example, the absolute value of a number is 5 means the point is 5 units from zero on the number line.

Definition 1.2:

The absolute value of a rational number 'x', denoted by |x| , is defined as:

$$|x| = \begin{cases} x \;\; if \; x \geq 0 \; and \\ -x \; if \; x < 0 \end{cases}$$

Example 1.7:

a. $|6| = 6$
b. $|0| = 0$
c. $|-15| = -(-15) = 15$

Example 1.8:

Simplify each of the following absolute value expressions.

a. $|8-3|$ b. $|-25+13|$ c. $|0-10|$

Solution:

a. since $8-3 = 5$ and $5 > 0$, we have $|8-3| = |5| = 5$

b. Since $-25+13 = -12$ and $-12 < 0$, we have
$|-25+13| = |-12| = -(-12) = 12$

c. Since $0-10 = -10$ and $-10 < 0$, we have
$|0-10| = |-10| = -(-10) = 10$

Equation involving absolute value

Definition 1.3:

An equation of the form **|x|** = a for any rational number a is called an absolute value equation.

Geometrically the equation $|x| = 8$ means that the point with coordinate x is 8 units from 0 on the number line. Obviously the number line contains two points that are 8 units from the origin, one to the right and the other to the left of the origin. Thus $|x| = 8$ has two solutions $|x| = 8$ and $x = -8$.

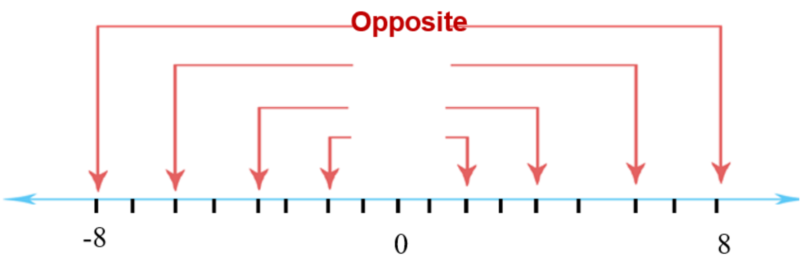


Figure1.11:

Note:

The solution of the equation $|x| = a$, or any rational number a has

i. Two solutions $x = a$ and $x = -a$ if $a > 0$
ii. One solution, $x = 0$ if $a = 0$ and
iii. No solution, if $a < 0$.

Example 1.9:

Solve the following absolute value equations.

a. $|x| = 13$ b. $|x| = 0$ c. $|x| = -6$

Solution:

a. $|x| = 13$

Since, $13 > 0$, $|x| = 13$ has two solutions: $x = 13$ and $x = -13$

b. $|x| = 0$

If $|x| = 0$, then $x = 0$

a. $|x| = -6$

Since, $-6 < 0$, $|x| = -6$ has no solution.

Exercise 1.3.

1. Complete the following table.

x	-7	$\frac{3}{5}$	0	$2\frac{5}{9}$	$-\frac{2}{3}$
|x|					

2. Find all rational numbers whose absolute values are given below

   a. 3.5   b. $\frac{4}{7}$ c. $\frac{11}{6}$ d. $3\frac{2}{5}$

3. Evaluate each of the following expressions.

   a. $|-5| + |5|$
   b. $|0| + 2\frac{1}{3}$
   c. $|-13| - |-8| + |7|$
   d. $\left|-\frac{1}{13}\right| + |-15| - 15$
   e. $|-8 + 5|$

4. Evaluate each of the following expressions for the given values of x and y.

   a. $5x - |x - 3|$, $x = -5$
   b. $|x| - x + 9$, $x = 3$
   c. $|x + y| - |x|$, $x = -3$ and $y = 6$
   d. $|x| + |y|$, $x = 5$ and $y = -10$
   e. $-3|x + 6|$, $x = -5$
   f. $\frac{|x| - |5y|}{|x + y|}$, $x = 4$ *and* $y = 8$

5. Solve the following absolute value equations.

   a. $|x| = 8$ b. $|x| = \frac{3}{5}$

Challenge problems

6. Solve the following absolute value equations.

   a. $|x + 4| = 10$
   b. $4|x+3| = 12$
   c. $3 - 2|x - 5| = 9$
   d. $|5x - 3| = \frac{5}{2}$
   e. $3|x - 5| = 3\frac{2}{3}$

1.2. Comparing and Ordering Rational numbers

Competency:

At the end of this sub-topic students should:

- Compare and order Rational numbers.

1.2.1. Comparing Rational numbers

In day to day activity, there are problems where rational numbers have to be compared. For instance, win and loss in games; positive and negative in temperature; profit and loss in trading etc.

Activity: 1.2.

Insert <, = or > to express the corresponding relationship between the following pairs of numbers.

a. 0 _______ 15

b. -3 _______ -5

c. 6.7_______6.89

d. $\frac{12}{3}$ _______ $\frac{8}{3}$

Comparing Decimals

A rational number $\frac{a}{b}$ can be expressed as a decimal number by dividing the numerator a by the denominator b.

Note:	Decimal numbers are compared in the same way as comparing other numbers: By comparing the different place values from left to right. That is, compare the integer part first and if they are equal, compare the digits in the tenths place, hundredths place and so on.

Example 1.10:

Compare the following decimal numbers.

a. 4.25 ______ 12.33

b. 15.52 ______ 15.05

c. 45.684 ______ 45.667

Solution:

a. Since 4 <12, then 4.25 < 12.33

b. The integer part of the two numbers are equal, then move to the tenth place and compare: 5> 0, then 15.52 > 15.05

c. Here the corresponding integer part and the tenth place numbers are equal, so we move to the hundredth place: 8 > 6. Thus 45.684 > 45.667

Comparing Fractions

Comparing fractions with the same denominator

If the denominators of two rational numbers are the same, then the number with the greater numerator is the greater number.

If $\frac{a}{b}$ and $\frac{c}{b}$ are a given rational numbers, $\frac{a}{b} > \frac{c}{b}$ if and only if $a > c$.

Example 1.11:

a. $\frac{15}{7} > \frac{13}{7}$ because 15 >13

b. $\frac{10}{3} < \frac{15}{3}$ because 10 < 15

Fractions that represent the same point on a number line are called Equivalent fractions. For any fraction $\frac{a}{b}$ and m is a rational number different from 0

($m \neq 0$), then $\frac{a}{b} = \frac{a}{b} \times \frac{m}{m}$.

Comparing fractions with different denominators

In order to compare any two rational numbers with different denominators, you can use either of the following two methods:

Method 1:

Change the fractions to equivalent fractions with the same denominators.

Step 1. Determine the LCM of the positive denominators.

Step 2. Write down the given rational numbers with the same denominators.

Step 3. Compare the numerators of the obtained rational numbers.

Example 1.12:

Compare the following pairs of rational numbers.

a. $\frac{3}{5}$ and $\frac{1}{2}$

b. $\frac{11}{16}$ and $\frac{7}{8}$

Solution:

a. To compare $\frac{3}{5}$ and $\frac{1}{2}$,

iv. Find the LCM of 5 and 2 which is 10.

v. Express the rational numbers with the same denominator 10.

$\frac{3}{5} = \frac{3}{5} \times \frac{2}{2} = \frac{6}{10}$ and $\frac{1}{2} \times \frac{5}{5} = \frac{5}{10}$

vi. Since $6 > 5$, $\frac{6}{10} > \frac{5}{10}$.

Therefore, $\frac{3}{5} > \frac{1}{2}$.

b. To compare $\frac{11}{16}$ and $\frac{7}{8}$

vii. Find the LCM of 16 and 8 which is 16.

viii. Express the rational numbers with the same denominator 16.

$\frac{11}{16} = \frac{11}{16} \times \frac{1}{1} = \frac{11}{16}$ and $\frac{7}{8} = \frac{7}{8} \times \frac{2}{2} = \frac{14}{16}$

Since $11 < 14$, $\frac{11}{16} < \frac{14}{16}$ Therefore, $\frac{11}{16} < \frac{7}{8}$

Method 2.

Cross- product method

Suppose $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers with positive denominators. Then

i. $\frac{a}{b} < \frac{c}{d}$, if and only if $ad < bc$

ii. $\frac{a}{b} > \frac{c}{d}$, if and only if $ad > bc$

iii. $\frac{a}{b} = \frac{c}{d}$, if and only if $ad = bc$

Example 1.13:

a. $\frac{5}{7} < \frac{8}{3}$ because $5 \times 3 = 15 < 7 \times 8 = 56$

b. $-\frac{12}{9} < \frac{6}{7}$ because $-12 \times 7 = -84 < 9 \times 6 = 54$

c. $\frac{9}{5} > \frac{11}{7}$ because $9 \times 7 = 63 > 5 \times 11 = 55$.

Comparing Rational numbers using number line

Note:	For any two different rational numbers whose corresponding points are marked on the number line, then the one located to the left is smaller.

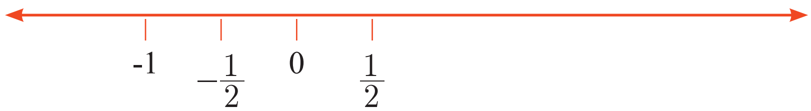


Figure1.12:

Thus, $-1 < -\frac{1}{2}$, $-\frac{1}{2} < 0$, $0 < \frac{1}{2}$

From the above fact, it follows that:

- Every positive rational number is greater than zero.
- Every negative rational number is less than zero.
- Every positive rational number is always greater than every negative rational number.
- Among two negative rational numbers, the one with the largest absolute value is smaller than the other. For instance, -45 < -23 because |-45| > |-23|

Exercise 1.4.

1. Which of the following statements are true and which are false?

   a. $-0.15 < 1.5$

   b. $2\frac{3}{5} > 3\frac{2}{5}$

   c. $\left|-\frac{3}{5}\right| < \frac{3}{5}$

   d. $\frac{12}{8} = \frac{10}{15}$

   e. $3\frac{5}{7} > \frac{21}{6}$

   f. $\frac{5}{17} > \frac{7}{18}$

   g. $6.53 < 6.053$

   h. $3\frac{4}{7} = \frac{25}{7}$

2. Insert (>, = or <) to express the corresponding relationship between the following pairs of numbers.

   a. $\frac{15}{9}$ ________ $\frac{18}{9}$

   b. $\frac{21}{12}$ ________ $-\frac{28}{16}$

   c. $\frac{8}{20}$ ________ 0.35

   d. $3\frac{5}{6}$ ________ $3\frac{7}{8}$

   e. $\left|-\frac{3}{10}\right|$ ________ $\frac{3}{10}$

   f. $\frac{12}{8}$ ________ $\frac{13}{9}$

3. From each pair of numbers, which number is to the left of the other on the number line?

   a. 3.5 , 7

   b. $\frac{6}{8}, \frac{5}{7}$

   c. $3\frac{2}{5}.\ 2\frac{5}{7}$

   d. $-9, \frac{13}{9}$

   e. $-\frac{1}{3}, -1.5$

4. k, n, m, x, y, z are natural numbers represented on a number line as follows:

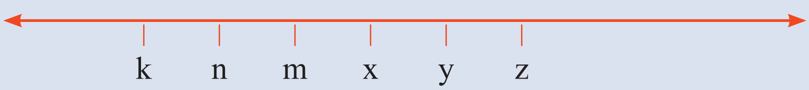


Figure1.13:

Compare the numbers using > or <.

   a. n__________x

   b. x___________y

   c. z___________n

   d. y___________k

1.2.2. Ordering Rational numbers

Ordering rational numbers means writing the given numbers in either ascending or descending order. Ordering a rational numbers of different denominators is a little bit like ordering distance measured in miles and kilometers, where we need all the distances to be in the same unit. For fractions, we need either to rewrite them in such a way that all have the same denominator or to convert them to decimals.

Example 1.14:

Arrange the following rational numbers in:

i. Increasing (ascending) order

a. $-25, 18, -45, 30, 28$

c. $2.5, 1.5, 3.21, 1.53, 2.05$

b. $\frac{3}{5}, \frac{4}{9}, \frac{7}{3}, \frac{5}{6}, \frac{11}{18}$

ii. Decreasing (descending) order

a. $-15, 45, 32, -23,$

c. $4.5, 5.17, 3.75, 4.75$

b. $\frac{7}{3}, 2\frac{4}{5}, \frac{5}{2}, \frac{8}{15},$

Solution:

i. a. $-25< -< -45 < 18 < 28 < 30$

b. To order these rational numbers, first change the fractions with the same denominator. Thus, $\frac{54}{90}, \frac{40}{90}, \frac{210}{90}, \frac{75}{90}$, and $\frac{55}{90}$.

Compare only the numerators: $40 < 54 < 55 < 75 < 210$

Therefore, the numbers in increasing order are $\frac{4}{9} < \frac{3}{5} < \frac{11}{18} < \frac{5}{6} < \frac{7}{3}$

c. $1.5 < 1.53 < 2.05 < 2.5 < 3.21$

ii. a. $45 > 32 > -15 > -23$

b. The numbers in the same denominator are $\frac{70}{30}, \frac{84}{30}, \frac{75}{30}, \frac{16}{30}$

Now compare only the numerators, $84 > 75 > 70 > 16$

Therefore, $2\frac{4}{5} > \frac{5}{2} > \frac{7}{3} > \frac{8}{15}$

c. $5.17 > 4.75 > 4.5 > 3.75$

Exercise 1.5.

1. Arrange the following rational numbers in ascending order.

   a. $\frac{4}{9}, \frac{3}{25}, \frac{11}{7}, -5\frac{2}{3}, \frac{23}{15}$

   b. 5.24, 8.13, 6.75, 12.42, -12.51

   c. $3, 9.2, \frac{5}{13}, 4\frac{6}{7}, 4.73, -\frac{11}{9}$

2. Arrange the following rational numbers in descending order.

   a. 13.72, 23.86, 15.02, 13.05

   b. $\frac{21}{12}, \frac{13}{16}, 2\frac{7}{5}, \frac{9}{7}, 2\frac{4}{9}$

   c. $3\frac{5}{6}$, , 3.75 , $\frac{18}{5}$, 4.23, 3.21

3. Samuel's science class is "growing plants under different conditions". The average plant growth during a week was 5.5cm.The table shows the difference from the average for some students' plants.

   a. Which student's plant growing more?

   b. Order the differences from lowest to highest.

Difference from Average Plant Growth

Student	Rahel	Kassa	Alemitu	Munir
Difference	$3\frac{1}{4}$	-2.3	1.7	$-1.\frac{7}{10}$

1.3. Operation and properties of Rational Numbers

Competencies:

At the end of this sub-unit students should:

- Add rational numbers.
- Subtract rational numbers.

- Multiply rational numbers
- Divide rational numbers.

1.3.1. Addition of rational numbers

Activity: 1.3.

1. Add the following numbers using a number line and show using arrows.
   a. 6 + -3 c. 2 + -4
   b. -9 + 5 d. 6 + -6
2. Find the sum of $\frac{3}{6}$ *and* $\frac{2}{6}$ using fraction bar.

Adding rational numbers with same denominators

To add two or more rational numbers with the same denominators, we add all the numerators and write the common denominator.

For any two rational numbers $\frac{a}{b}$ and $\frac{c}{b}$, $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$

Example 1.15:

Find the sum of the following rational numbers

a. $\frac{3}{5}+\frac{6}{5}$

b. $\frac{8}{7}+\frac{5}{7}+\frac{6}{7}$

c. $-\frac{3}{4}+\frac{11}{4}$

d. $2\frac{1}{3}+\frac{4}{3}+1$

Solution:

a. $\frac{3}{5}+\frac{6}{5}=\frac{3+6}{5}=\frac{9}{5}$

b. $\frac{8}{7}+\frac{5}{7}+\frac{6}{7}=\frac{8+5+6}{7}=\frac{19}{7}$

c. $-\frac{3}{4}+\frac{11}{4}=\frac{-3+11}{4}=\frac{8}{4}=2$

d. $2\frac{1}{3}+\frac{4}{3}+1=\frac{7}{3}+\frac{4}{3}+\frac{3}{3}=\frac{7+4+3}{3}=\frac{14}{3}$

Adding rational numbers with different denominators

To find the sum of two or more rational numbers which do not have the same denominator, we follow the following steps:

i. Make all the denominators positive.
ii. Find the LCM of the denominators of the given rational numbers.
iii. Find the equivalent rational numbers with common denominator.
iv. Add the numerators and take the common denominator.

Example 1.16:

Find the sum of the following rational numbers.

a. $\frac{3}{7} + \frac{5}{4}$ b. $\frac{11}{9} + \frac{5}{6} + \frac{3}{4}$

Solution:

a. The LCM of 7 and 4 is 28.

Then write the fraction as a common denominator 28.

$$\frac{3}{7} \times \frac{4}{4} = \frac{12}{28} \text{ and } \frac{5}{4} \times \frac{7}{7} = \frac{35}{28}$$

$$\frac{3}{7} + \frac{5}{4} = \frac{12}{28} + \frac{35}{28} = \frac{12+35}{28} = \frac{47}{28}$$

b. The LCM of 9 , 6 and 4 is 36.

Then, write the fraction as a common denominator 36.

$$\frac{11}{9} \times\times \frac{4}{4} = \frac{44}{36} \text{ , } \frac{5}{6} \times \frac{6}{6} = \frac{30}{36} \text{ and } \frac{3}{4} \times \frac{9}{9} = \frac{27}{36}$$

$$\frac{11}{9} + \frac{5}{6} + \frac{3}{4} = \frac{44}{36} + \frac{30}{36} + \frac{27}{36} = \frac{44+30+27}{36} = \frac{101}{36}$$

Note:	Always reduce your final answer to its lowest term.

Now you are going to discover some efficient rules for adding any two rational numbers.

Rule 1: To find the sum of two rational numbers where both are negatives:

i. Sign : Negative (−)
ii. Take the sum of the absolute values of the addends.
iii. Put the sign in front of the sum.

Example 1.17:

Perform the following operation:

$$-\frac{8}{6} + (-\frac{11}{8})$$

Solution:

$-\frac{8}{6} + (-\frac{11}{8})$

i. Sign (−)

ii. $\left|-\frac{8}{6}\right| + \left|-\frac{11}{8}\right| = \frac{8}{6} + \frac{11}{8} = \frac{65}{24}$

Therefore, $-\frac{8}{6} + (-\frac{11}{8}) = -\frac{65}{24}$

Rule 2: To find the sum of two rational numbers, where the signs of the addends are different, are as follows:

i. Take the sign of the addend with the greater absolute value.
ii. Take the absolute values of both numbers and subtract the addend with smaller absolute value from the addend with greater absolute value.
iii. Put the sign in front of the difference.

Example 1.18:

Perform the following operation

a. $-\frac{9}{4} + \frac{5}{4}$ b. $-\frac{5}{6} + \frac{3}{4}$

Solution:

a. $-\frac{9}{4} + \frac{5}{4}$

i. Sign(−) because $\left|-\frac{9}{4}\right| > \left|\frac{5}{4}\right|$

ii. Take the difference of the absolute values:

$$\left|-\frac{9}{4}\right| - \left|\frac{5}{4}\right| = \frac{9}{4} - \frac{5}{4} = \frac{4}{4} = 1$$

Therefore, $-\frac{9}{4} + \frac{5}{4} = -\frac{4}{4} = -1$

b. $-\frac{5}{6} + \frac{3}{4}$

i. Sign(−) because $\left|-\frac{5}{6}\right| > \left|\frac{3}{4}\right|$

ii. Take the difference of the absolute values:

$$\left|-\frac{5}{6}\right| - \left|\frac{3}{4}\right| = \frac{5}{6} - \frac{3}{4} = \frac{1}{12}$$

Therefore, $-\frac{5}{6} + \frac{3}{4} = -\frac{1}{12}$

Example 1.19:

Find the sum of $\frac{5}{8}$ and $\frac{2}{8}$ using fraction bar.

Solution:

Divide the fraction bar into 8 equal parts. Now shade 5 of them with gray color and 2 of them with green color as shown in the figure below:

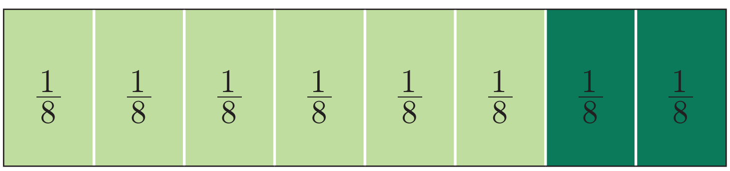


Figure1.14:

The shaded part within gray color represents $\frac{5}{8}$ and within green color represents $\frac{2}{8}$ of the whole fraction bar. The shaded part in both colors is 7 of eight equal parts. This shows that $\frac{5}{8} + \frac{2}{8} = \frac{7}{8}$

Example 1.20:

Find the sum of $\frac{1}{6}$ and $\frac{1}{2}$.

Solution:

Divide the fraction bar into 6 equal parts and shade one of them, which represents $\frac{1}{6}$. Similarly, Shade the other three parts in different colors which represents $\frac{3}{6} = \frac{1}{2}$.

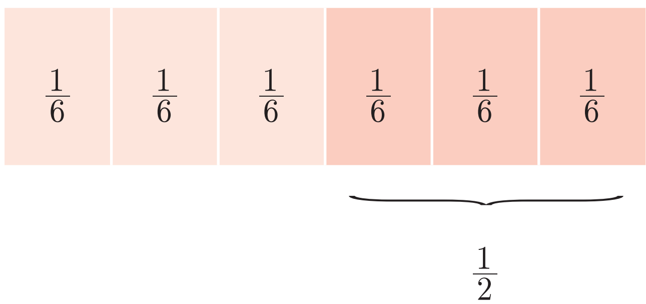


Figure1.15:

How many parts of the fraction bar shaded? 4 parts of the 6 equal parts.

This implies that $\frac{1}{6} + \frac{1}{2} = \frac{1}{6} + \frac{3}{6} = \frac{4}{6} = \frac{2}{3}$

Properties of Addition of Rational Numbers

For any rational numbers a, b and c the following properties of addition holds true:

a. Commutative: $a + b = b + a$

b. Associative: $a + (b + c) = (a + b) + c$

c. **Properties of 0:** $a + 0 = a = 0 + a$

d. Properties of opposites : $a + (-a) = 0$

Example 1.21:

Using properties of addition find the sum: $\frac{1}{3} + \frac{3}{5} + \frac{7}{6}$

Solution:

$\frac{1}{3}+\frac{3}{5}+\frac{7}{6} = (\frac{1}{3}+\frac{3}{5})+\frac{7}{6}$ -------- Associative property

$= (\frac{5+9}{15})+\frac{7}{6}$

$= \frac{14}{15}+\frac{7}{6}$

$= \frac{28+35}{30}$

$= \frac{63}{30}$

$= \frac{21}{10}$

Similarly, $\frac{1}{3}+\frac{3}{5}+\frac{7}{6} = \frac{1}{3}+(\frac{3}{5}+\frac{7}{6})$ ------ Associative property

$= \frac{1}{3}+(\frac{18+35}{30})$

$= \frac{1}{3}+\frac{53}{30}$

$= \frac{10+53}{30}$

$= \frac{63}{30}$

$= \frac{21}{10}$

Therefore, $(\frac{1}{3}+\frac{3}{5})+\frac{7}{6} = \frac{1}{3}+(\frac{3}{5}+\frac{7}{6})$

Exercise 1.6.

1. Find the sum:
   - *a.* $\frac{13}{5}+\frac{21}{5}$
   - *b.* $\frac{5}{6}+\frac{3}{8}$
   - *c.* $\frac{3}{5}+2\frac{3}{5}$
   - *d.* $\frac{1}{23}+\frac{3}{8}+3\frac{5}{6}$
   - *e.* $|-\frac{2}{5}+\frac{3}{8}|+|\frac{4}{7}+\frac{2}{7}|$
2. In the city where I live, the temperature on an outdoor thermometer on Monday was 23.72°c. The temperature on Thursday was 3.23°c more than that of Monday. What was the temperature on Thursday?

1.3.2. Subtraction of rational numbers

Activity: 1.4.

1. Can you express subtraction of rational numbers in the form of addition?
2. Are the commutative and associative properties holds true in subtraction of rational numbers?

The process of subtraction of rational numbers is the same as that of addition. Subtraction of any rational numbers can be explained as the inverse of addition: That is, for two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, subtracting $\frac{c}{d}$ from $\frac{a}{b}$ means adding the negative of $\frac{c}{d}$ to $\frac{a}{b}$. Thus $\frac{a}{b} - \frac{c}{d} = \frac{a}{b} + (-\frac{c}{d})$

Example 1.22:

Compute the following difference.

a. $\frac{7}{9} - \frac{4}{3}$ b. $\frac{3}{7} - (-\frac{5}{9})$ c. $-\frac{1}{12} - \frac{9}{8}$

Solution:

a. $\frac{7}{9} - \frac{4}{3} = \frac{7}{9} + (-\frac{4}{3})$

$= \frac{7 + (-12)}{9}$

$= -\frac{5}{9}$

b. $\frac{3}{7} - (-\frac{5}{9}) = \frac{3}{7} + \frac{5}{9}$

$= \frac{27+35}{63}$

$= \frac{62}{63}$

c. $-\frac{1}{12} - \frac{9}{8} = -\frac{1}{12} + (-\frac{9}{8})$

$= \frac{-2 + (-27)}{24}$

$= -\frac{29}{24}$

Note:
i. The difference of two rational numbers is always a rational number.
ii. Addition and subtraction are inverse operations of each other.

Example 1.23:

Find the difference of $\frac{7}{9}-\frac{2}{3}$ using fraction bar.

Solution:

Divide the fraction bar in to 9 equal parts and shade 7 parts of them. 6 parts of the 9 equal parts $\frac{6}{9}$ represent $\frac{2}{3}$ of the fraction bar.

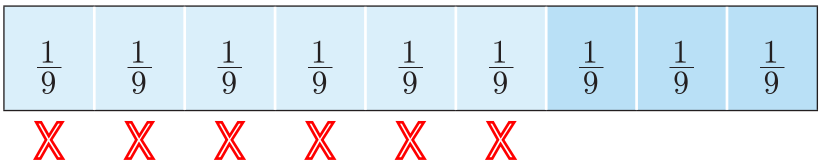


Figure1.16:

How many parts of the shaded part is unmarked? One part. This implies that

$$\frac{7}{9}-\frac{2}{3}=\frac{7}{9}-\frac{6}{9}=\frac{1}{9}$$

Exercise 1.7.

3. Find the difference of each of the following

   a. $4\frac{5}{6}-2\frac{3}{4}$
   b. $-5.3-3.45$
   c. $\frac{6}{13}-|-\frac{7}{13}|$
   d. $|-\frac{5}{7}|-|\frac{3}{4}|$
   e. $-32.24-|-32.24|$
   f. $-3\frac{2}{5}-2\frac{3}{7}$

4. Evaluate the following expressions:

   a. $y-(\frac{2}{7}+5),\ when\ y=\frac{9}{4}$
   b. $15-(-y-\frac{4}{9}),\ when\ y=7$

5. From a rope 23 m long, two pieces of lengths $\frac{12}{7}$ m and $\frac{7}{4}$ m are cut off. What is the length of the remaining rope?

6. A basket contains three types of fruits, apples, oranges and bananas, weighing $\frac{58}{3}$ kg in all. If $\frac{18}{7}$ kg be apples, $\frac{11}{9}$ kg be oranges and the rest are bananas. What is the weight of the bananas in the basket?

1.3.3. Multiplication of rational numbers

Activity: 1.5.

Multiply the following rational numbers

a. $\frac{5}{7} \times \frac{8}{11}$

b. $-\frac{9}{7} \times \frac{4}{5}$

c. $\frac{2}{3}(\frac{5}{9} \times (-\frac{3}{5}))$

d. $3\frac{2}{5} \times 2\frac{4}{7}$

e. $-\frac{3}{8} \times (-\frac{2}{3})$

To multiply two or more rational numbers, we simply multiply the numerator with the numerator and the denominator with the denominator. Finally reduce the final answer to its lowest term if it is.

Example 1.24:

Find the product of $\frac{1}{2}$ and $\frac{3}{4}$ using grids model.

Solution:

a. a. model $\frac{1}{2}$ by shading half of a grid

b. b. use a different color to shade $\frac{3}{4}$ of the same grid

i. Divide the grid in to 2 columns.

ii. Shade 1 column to show $\frac{1}{2}$

iii. Divide the grid in to 4 rows

iv. Shade 3 rows to show $\frac{3}{4}$

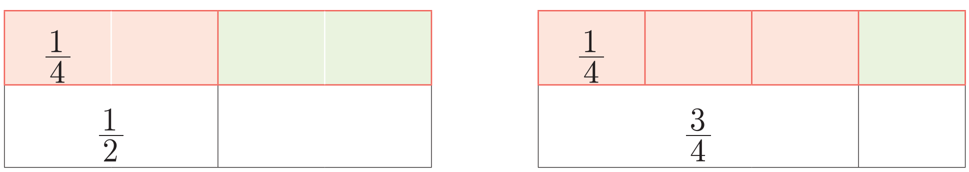


Figure1.17:

a. Determine what fraction of the grid is shaded with both colors.

There are 8 equal parts, and 3 of the part are shaded with both colors. The fraction shaded with both colors is $\frac{3}{8}$.

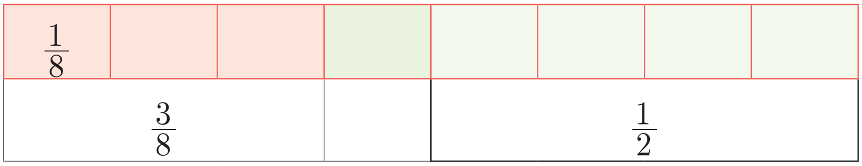


Figure1.18:

The section of the grid shaded with both colors shows 3 part of $\frac{1}{2}$ when $\frac{1}{2}$ is divided into 4 equal parts. In other words $\frac{3}{4}$ of $\frac{1}{2}$, or $\frac{3}{4} \times \frac{1}{2}$

Therefore, $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$

Note:	The product of two rational numbers with different signs can be determine in three steps b. Decide the sign of the product; it is "– ". c. Take the product of the absolute value of the numbers. d. Put the sign in front of the product.

Example 1.25:

Find the product

a. $\frac{3}{4} \times \frac{7}{9}$ b. $\frac{4}{9} \times (-\frac{5}{2})$ c. $2\frac{5}{7} \times (-\frac{3}{2})$

Solution:

a. $\frac{3}{4} \times \frac{7}{9} = \frac{3 \times 7}{4 \times 9} = \frac{21}{36} = \frac{7}{12}$

b. $\frac{4}{9} \times (-\frac{5}{2})$

i. Sign (–)

ii. Multiply the absolute value

$$\left|\frac{4}{9}\right| \times \left|-\frac{5}{2}\right| = \frac{4}{9} \times \frac{5}{2} = \frac{20}{18} = \frac{10}{9}$$

Therefore, $\frac{4}{9} \times (-\frac{5}{2}) = -\frac{20}{18} = -\frac{10}{9}$

c. $2\frac{5}{7} \times (-\frac{3}{2})$

First change the mixed number to improper fraction.

$2\frac{5}{7} = 2 + \frac{5}{7} = \frac{19}{7}$, then

i. Sign (−)

ii. Multiply the absolute value

$$\left|\frac{19}{7}\right| \times \left|-\frac{3}{2}\right| = \frac{19}{7} \times \frac{3}{2} = \frac{57}{14}$$

Therefore, $2\frac{5}{7} \times (-\frac{3}{2}) = -\frac{57}{14}$

Note:	The product of two negative rational numbers is a positive rational number.

Example 1.26:

Find the product: $-\frac{2}{5} \times (-\frac{7}{3})$

Solution: $-\frac{2}{5} \times (-\frac{7}{3}) = \frac{2}{5} \times \frac{7}{3} = \frac{14}{15}$

The following table 1.1 summarizes the facts about product of rational numbers.

Table 1.1

The two factors	The product	Example
Both positive	Positive	$\frac{2}{3} \times 5 = \frac{10}{3}$
Both negative	Positive	$(-\frac{5}{4}) \times (-\frac{7}{3}) = \frac{35}{12}$
Of opposite sign	Negative	$-3 \times 5 = -15$
One or both 0	Zero	$-\frac{7}{9} \times 0 = 0$

Properties of multiplication of rational numbers

For any rational numbers a, b and c ,the following properties of multiplication holds true:

a. **Commutative:** $a \times b = b \times a$

b. **Associative:** $a \times (b \times c) = (a \times b) \times c$

c. **Distributive:** $a \times (b + c) = a \times b + a \times c$
d. **Property of 0:** $a \times 0 = 0 = 0 \times a$
e. **Property of 1:** $a \times 1 = a = 1 \times a$

Example 1.27:

Using the properties of multiplication, find the following products.

a. $\frac{3}{5} \times \frac{6}{7}$

b. $\frac{2}{3} \times \frac{4}{5} \times \frac{5}{2}$

c. $\frac{2}{5} \times (\frac{3}{7} + \frac{4}{3})$

d. $(\frac{8}{9} \times \frac{5}{4}) \times 0$

e. $2\frac{3}{5} \times 1$

Solution:

a. $\frac{3}{5} \times \frac{6}{7} = \frac{6}{7} \times \frac{3}{5} = \frac{18}{35}$

b. $(\frac{2}{3} \times \frac{4}{5}) \times \frac{5}{2}$ ….. associative property … $\frac{2}{3} \times (\frac{4}{5} \times \frac{5}{2})$

$= \frac{8}{15} \times \frac{5}{2}$ $\qquad = \frac{2}{3} \times \frac{20}{10}$

$= \frac{40}{30}$ $\qquad = \frac{40}{30}$

$= \frac{44}{33}$ $\qquad = \frac{4}{3}$

c. $\frac{2}{5} \times (\frac{3}{7} + \frac{4}{3})$

$= \frac{2}{5} \times \frac{3}{7} + \frac{2}{5} \times \frac{4}{3}$ ….. distributive property

$= \frac{6}{35} + \frac{8}{15}$

$= \frac{74}{105}$

d. $(\frac{8}{9} \times \frac{5}{4}) \times 0 = 0$……..property of 0

e. $2\frac{3}{5} \times 1 = 2\frac{3}{5}$ ………property of 1

Example 1.28:

Find the cost of $\frac{35}{9}$m of cloth, if the cost of a cloth per meter is Birr $\frac{162}{4}$.

Solution:

Total cost $= \frac{35}{9} \times \frac{152}{4} - \frac{5670}{36} = Birr\ \frac{315}{2} =$ Birr 157.5

Note: To get the product with three or more factors, we use the following properties:

1. The product of an even number of negative factors is positive.
2. The product of an odd number of negative factors is negative.
3. The product of a rational number with at least one factor 0 is zero.

Exercise 1.8.

1. Determine the product
   a. $\frac{5}{8} \times \frac{3}{4}$
   b. $\frac{3}{7} \times (-\frac{3}{4})$
   c. $-\frac{8}{7} \times (-\frac{3}{5})$
   d. $2\frac{3}{5} \times 4\frac{2}{3}$
   e. $-\frac{2}{3} \times \frac{5}{4} \times \frac{3}{7}$
   f. $-3(5x+5)$, *when* $x = \frac{3}{4}$
2. An airplane covers 1250 km in an hour. How much distance will it cover in $\frac{23}{6}$ hours?

1.3.4. Division of Rational Numbers

Activity: 1.6.

1. How many groups of $\frac{3}{4}$ are in 12?

2. How many groups of $\frac{2}{5}$ are in $3\frac{3}{5}$?

3. Use grids to model $3\frac{1}{3} \div \frac{2}{3}$

Example 1.29:

Find the quotient by dividing $4\frac{1}{3}$ by $\frac{2}{3}$ using grid model.

Solution:

Since $4\frac{1}{3} = 4 + \frac{1}{3}$, divide each 5 grids in to 3 equal parts.

Shade 4 grids and $\frac{1}{3}$ of a fifth grid to represent $4\frac{1}{3}$ and

Divide the shaded grids in to equal groups of 2.

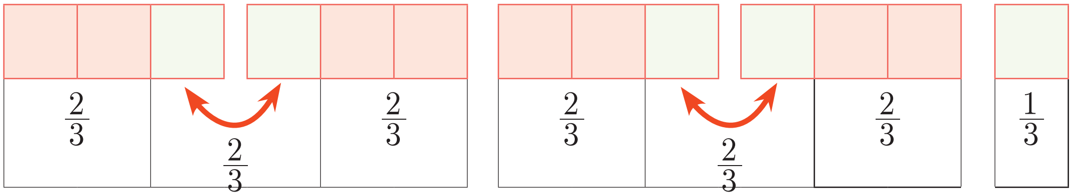


Figure1.19:

There are 6 groups of $\frac{2}{3}$, with in $\frac{1}{3}$ left over. This piece is $\frac{1}{2}$ of a group of $\frac{2}{3}$.

Thus, there are $6 + \frac{1}{2}$ groups of $\frac{2}{3}$ in $4\frac{1}{3}$.

$$4\frac{1}{3} \div \frac{2}{3} = 6 + \frac{1}{2} = \frac{13}{2}$$

Note:

- $a \div b$ is read as a is divided by b.
- In $a \div b = c$ is called the quotient, a is called the dividend and b is called the divisor.
- The quotient $a \div b$ is also denoted by $\frac{a}{b}$.
- If a, b and c are integers, $b \neq 0$ and $a \div b = c$, if and only if $b \times c = a$.

Rules for Division of Rational numbers

When dividing rational numbers:

1. Determine the sign of the quotient:

   a. If the sign of the dividend and the divisor are the same, then sign of the quotient is (+).

   b. If the sign of the dividend and the divisor are different, the sign of the quotient is (−).

2. Determine the value of the quotient by dividing the absolute value of the dividend by the divisor.

Example 1.30:

$$\frac{-324}{-18} = \frac{324}{18} = 18$$

Note: For any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc} \quad \text{(where } c \neq 0)$$

Example 1.31:

Determine the quotient:

a. $\frac{6}{14} \div \frac{3}{7}$ b. $-3 \div \frac{9}{15}$ c. $-\frac{5}{7} \div -4$

Solution:

a. $\frac{6}{14} \div \frac{3}{7} = \frac{6}{14} \times \frac{7}{3} = \frac{6 \times 7}{14 \times 3} = \frac{42}{42} = 1$

b. $-3 \div \frac{9}{15} = -\frac{3}{1} \div \frac{9}{15} = -\frac{3}{1} \times \frac{15}{9} = -\frac{45}{9} = -5$

c. $-\frac{5}{7} \div -4 = -\frac{5}{7} \times -\frac{1}{4} = \frac{5}{28}$

Note: For any rational number $\frac{a}{b}$ where $a \neq 0$;

$\frac{a}{b} \times \frac{b}{a} = \frac{ab}{ba} = 1$, Then $\frac{b}{a}$ is called the reciprocal of $\frac{a}{b}$.

Exercise 1.9.

1. Determine the quotient

   a. $\frac{5}{8} \div \frac{3}{4}$ c. $-\frac{8}{9} \div (-\frac{5}{3})$

   b. $-\frac{3}{5} \div \frac{6}{7}$ d. $2\frac{3}{5} \div (-\frac{4}{3})$

2. Rahel made $\frac{3}{4}$ of a pound of trail mix. If she puts $\frac{3}{8}$ of a pound into each bag, how many bags can Rahel fill?

1.4. Real life applications of rational numbers

Rational numbers used to express many day –to- day real life activities. For instance, to share something among friends, to calculate interest rates on loans and mortgages, to calculate interest on saving accounts, to determine shopping discounts, to calculate prices, to prepare budgets, etc. So, in this sub-topic we will discuss some of them.

1.4.1. Application in sharing something among friends

Rational numbers are used in sharing and distributing something among a group of friends.

Example 1.32:

There are four friends and they wanting to divide a cake equally among themselves. Then, the amount of cake each friend will get is one fourth of the total cake.

Example 1.33:

Three brothers buy sugarcane. Their mother says that she will take over a fifth of the sugarcane. The brothers share the remaining sugarcane equally. What fraction of the original sugarcane does each brother get?

Solution:

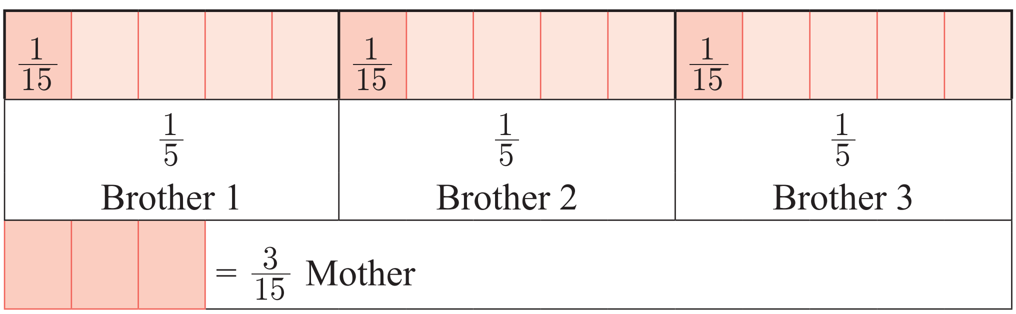


Figure1.20:

Each brother gets a big piece $\frac{1}{5}$ and divides one big piece in to three equal parts, $\frac{1}{5} \div 3$, which is $\frac{1}{15}$. so each brother gets $\frac{1}{5} + \frac{1}{15} = \frac{3}{15} + \frac{1}{15} = \frac{4}{15}$

1.4.2. Application in calculating interest and loans

Simple interest

Interest is a payment for the use of money or interest is the profit return on investment. Interest can be paid on money that is borrowed or loaned. The borrower pays interest and the lender receives interest. The money that is borrowed or loaned is called the principal (P). The portion paid for the use of money is called the interest (I). The length of time that money is used or for which interest is paid is called time (T). The interest paid on the original principal during the whole interest periods is called simple interest. Interest can be calculated by: I = PRT

Example 1.34:

Abebe borrowed Birr 21100 from CBE five months ago. When he first borrowed the money, they agreed that he would pay to CBE 15% simple interest. If Abebe pays to it back today, how much interest does he owe to it?

Solution:

Given	Required
P = **B**irr 21100	I = ?
R = 15%	
I = PRT	

$I = \text{Birr } 21100 \times 15\% \times \frac{5}{12}$, Where, T = 5 months = $\frac{5}{12}$ *years*

$I = \text{Birr } 21100 \times 0.15 \times \frac{5}{12}$

$I = \text{Birr } 3165 \times \frac{5}{12}$

$I = \text{Birr } 1318.75$

Therefore, Abebe pay an additional 1318.75 Birr of simple interest as per their agreement.

Example 1.35:

What principal would give Birr 250 interest in $2\frac{1}{2}2\frac{1}{2}$ years at a rate of 10%?

Solution:

Given	Required
I = Birr 250	P =?
R = 10%	
T = $2\frac{1}{2}$ years = 2.5 years	
I = PRT	

$$P = \frac{I}{RT} = \frac{Birr\ 250}{10\% \times 2.5}$$

$$P = \frac{Birr250}{0.1 \times 2.5} = Birr\ 1000$$

Exercise 1.10.

1. If Birr 1200 is invested at 10% simple interest per annum, then What is simple interest after 5 years?
2. What principal will bring Birr 637 interest at a rate of 7% in 2 years?
3. Find the simple interest rate for a loan where Birr 6000 is borrowed and the amount owned after 5 months is Birr 7500.

SUMMARY FOR UNIT 1

1. A rational number is a number that can be written as $\frac{a}{b}$ where a and b are integers and $b \neq 0$. The set of rational numbers is denoted by $\mathbb{Q}$.
2. Rational numbers can represent on a number line.
3. The absolute value of a rational number 'x', denoted by $|x|$, is defined as:

$$|x| = \begin{cases} x \ \ if\ x > 0, \\ 0\ if\ x = 0\ and \\ -x\ if\ x < 0 \end{cases}$$

4. Rational numbers can be compared
   i. By changing to decimal numbers
   ii. By making the same denominators
   iii. By using cross product
5. Subtraction of any rational numbers can be explained as the inverse of addition.
6. The sum of two opposite rational numbers is 0.
7. Rules of signs of Addition
   a. The sum of two negative rational numbers is negative.
   b. The sum of negative and positive rational numbers is takes the sign of the greater absolute value.
8. Rules of signs of Multiplication
   a. The product of two negative rational numbers is positive.
   b. The product of negative and positive rational numbers is negative.
9. Rules of signs for division.

   Let a and b be rational numbers:

   a. positive divided by negative equals negative.
   b. negative divided by positive equals negative.
   c. negative divided by negative equals positive.

REVIEW EXERCISE FOR UNIT 1

1. Which of the following statements are true?
   a. $\frac{5}{3} > -\frac{8}{11}$
   b. $\frac{5}{7} > \frac{23}{7} > \frac{19}{6}$
   c. $\left|2\frac{3}{5}\right| = \left|-2\frac{3}{5}\right|$
   d. $\frac{5}{6} < 3\frac{4}{5} < \frac{2}{7}$
2. Solve each of the following absolute value equations.
   a. $|x| = 7$
   b. $|x - 5| = 4$
   c. $|2x + 3| = 5$
   d. $2|3x - 4| = 6$
   e. $5 + 3|x - 3| = 2$

3. If $x = -8$ and $y = 4$, then fined $\frac{3|x-5|-|4y|}{|x+y|}$

4. Find the sum

a. $\frac{3}{4}+\frac{9}{7}+2\frac{3}{5}$

b. $-2\frac{1}{3}+1\frac{4}{7}$

c. $3.35+2\frac{3}{7}$

5. Find the difference

a. $\frac{2}{7}-\frac{3}{8}$

b. $-2\frac{6}{7}-\frac{13}{9}$

6. Determine the product

a. $2\frac{3}{7}\times\frac{1}{4}$

b. $2.34\times\frac{7}{6}$

c. $-3\frac{3}{5}\times 1\frac{2}{3}\times(-\frac{3}{4})$

7. Determine the quotient

a. $\frac{3}{4}\div(-\frac{2}{7})$

b. $-3\frac{5}{8}\div 2\frac{3}{10}$

c. $2\frac{3}{7}\div 2.3$

8. Simplify the following expressions.

a. $\frac{3}{5}+\frac{2}{7}(\frac{4}{5}+\frac{6}{7})$

b. $\frac{4}{5}\div(\frac{5}{9}-\frac{5}{7})$

c. $\frac{\frac{1}{6}\div(\frac{1}{8}+\frac{4}{5})}{\frac{5}{8}+\frac{1}{8}(\frac{2}{5}\div\frac{8}{5})}$

9. Eleni baked a batch of 32 cupcakes and iced 24 of them. What fractions of cupcakes were iced?

10. How long will take Birr 500 to get Birr 50 simple interest at rate of 9.5%?

Unit 2

2. SQUARES, SQUARE ROOTS, CUBES AND CUBE ROOTS

Learning Outcomes:

At the end of this unit, learners will able to:

- Understand the notion square and square roots, and cubes and cubes roots.
- Determine the square of numbers
- Determine the square roots of the perfect square numbers
- Extract the approximate square roots of numbers by using the numerical table and scientific calculator
- Determine the cube of numbers
- Extract the cube roots of perfect cubes
- Apply squares, square roots, cubes and cube roots in the real-life situation

Main content

3.1. Squares and Square Roots

3.2. Cubes and Cube roots

3.3. Applications on squares, square roots, cubes and cube roots

- Summary
- Review Exercise

Introduction

What you had learnt in the previous grades about multiplication will be used in this unit to describe special products known as squares and cubes of the given numbers. For instance, in a real-life application when you want to find a new apartment, you need to know the size of it. Newspaper and online advertisements usually give the dimensions of the apartment by only listing its area, such as 625625 square meters. This can be hard to you use the concepts of square roots and convert this number in the form of $25 \times 2525 \times 25$ square meters. It gives better ideas of the apartment. The following sub-topics will give a brief explanation about squares and cubes.

2.1. Squares and Square roots

In this sub-topic you will learn about raising a given number to the power of "2" and extracting square roots of some perfect squares.

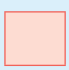
2.1.1. Square of a rational number

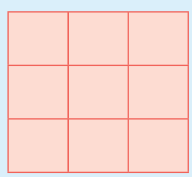
Competency: **At the end of this sub-topic, students should:**

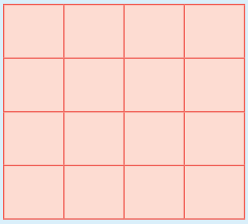
- Calculate the square of a number

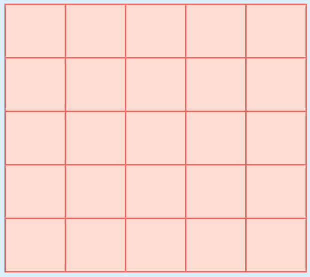
Activity: 2.1.

1. How many numbers of squares do you have?

a. 

c. 

a. 

d. 

2. Find the products of each of the following

i. 4×5 ____________

iii. 2.5×6______

ii. $3\frac{1}{2} \times 2.5$ ___________

3. Compute the products for each of the following.

a. $6 \times 6 =$ ___________

c. 0.5×0.5 _____________

b. $10 \times 10 =$ ___________

Definition 2.1

The process of multiplying a number by itself is called squaring of a number.

Example 2.1.

a. $1x1 = 1 = 1^2$

c. $4x4 = 16 = 4^2$

b. $2x2 = 4 = 2^2$

d. $5x5 = 25 = 5^2$

Note: If the number “a“ to be multiplied by itself, then the product is usually written a^2 and read as:

- a squared or
- The square of a or
- a to the power of 2

a)

Example2.2:

Read the following numbers

a. 2^2 b. 6^2 c. 10^2

Solution

b. 2^2 read as 2 squared or the square of 2 and 2 to the power of 2

c. 6^2 read as 6 squared or the square of 6 and 6 to the power of 2

d. 10^2 read as 10 squared or the square of 10 and 10 to the power of 2

Example 2.3:

Find the square of each of the following numbers

a. 8 c. 20

b. $\frac{4}{3}$ d. $-\frac{10}{16}$

Solution:

a. $8^2 = 8\ 8^2 = 8 \times 8 = 64$ c. $20^2 = 20 \times 20 = 400$

b. $\left(\frac{4}{3}\right)^2 = \frac{4}{3} \times \frac{4}{3} = \frac{16}{9}$ d. $\left(-\frac{10}{16}\right)^2 = -\frac{10}{16} \times -\frac{10}{16} = \frac{100}{256}$

Group Work 2.1

Discuss the following patterns with your groups:

1 [first odd number] = 1 = 1^2

1 + 3 [sum of first two odd numbers] = 4 = 2^2

1 + 3 + 5 [sum first three odd numbers] = 9 = 3^2

1 + 3 + 5 + 7 [sum first four odd numbers] = 16 = 4^2

⋮

1+3+5+7+9+ ..... +19 [sum of first ten odd numbers] = 100 = 10^2

a. What is the sum of the first 15 odd numbers?

b. What is the sum of the first n odd numbers?

c. What do you generalize?

From the above group work, we generalize that the sum of the first n odd natural numbers is n^2.

Note:	There is a difference between a^2 and $2a$ i. $a^2 = a \times a$ ii. $2a = a + a$

Consider the following examples to see the difference

a. $10^2 = 10 \times 10 = 100$ *while* $10 \times 2 = 20$

b. $0.4^2 = 0.4 \times 0.4 = 0.16$ *while* $0.4 \times 2 = 0.8$

In general, $a^2 \neq 2a$ for any rational number a.

Definition 2.2

A rational number x is called a perfect square if and only if

$x = m^2$ for some $m \in \mathbb{Q}$

Example 2.4:

$1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, $5^2 = 25$, $6^2 = 36$. Thus, 1, 4, 9, 16, 25 and 36 are perfect squares.

Note:	i. The square of rational numbers is also rational numbers. ii. 0 x 0 = Therefore, $0^2 = 0$ iii. For any rational numbers a and b, $(ab)^2 = a^2b^2$ iv. For any rational number a and b, $(\frac{a}{c})^2 = \frac{a^2}{b^2}$ where $b \neq 0$

Exercise 2.1.

1. Determine whether each of the following statements is true or false.

   a. $10^2 = 10 \times 10$

   b. $8^2 = 8 \times 2$

   c. $x^2 = 2x$, *where* $x \in \mathbb{Q}$

   d. $-60^2 = 3600$

   e. $(-30)^2 = 900$

2. Find x^2 in each of the following rational numbers

a. $x = 6$

b. $x = -20$

c. $x = 3\frac{1}{4}$

d. $x = -\frac{5}{4}$

e. $x = 0.03$

f. $x = 4.5$

g. $x = \frac{12}{4}$

3. Lists out a perfect square numbers from the given list.

$4, 9, 12, 16, 18, 24, 36, 1.6, 0.04, \frac{1}{4}, \frac{1}{121}, 0.01, 225$

4. Explain whether the numbers are a square number or not.

a. 144 b) 201 c) 324

5. Which of the following are the squares of even numbers?

a. 196 b) 441 c) 400 d) 324 e) 625

6. Which of the following are the squares of odd numbers?

a. 121 b) 225 c) 196 d) 484 e) 529

7. Evaluate

i. $(38)^2 - (37)^2$

ii. $(75)^2 - (74)^2$

iii. $(92)^2 - (91)^2$

iv. $(105)^2 - (104)^2$

v. $(141)^2 - (140)^2$

vi. $(218)^2 - (217)^2$

What do you conclude from the pattern?

8. Express 64 as the sum of the first eight odd numbers.

Theorem 2.1: Existence theorem

For any rational number x , there is a rational number $y \geq 0$ such that $x^2 = y$

Example 2.5:

Find the square of the following rational numbers by using the existence theorem.

a. $x = 12$

b. $x = 0.040$

c. $x = \frac{1}{2}$

d. $x = \frac{1}{4}$

Solution:

a. $x = 12$, then $y = x^2 = 12^2 = 144$

b. $x = 0.04$, then $y = x^2 = 0.04^2 = 0.0016$

c. $x = \frac{1}{2}$, then $y = x^2 = (\frac{1}{2})^2 = \frac{1}{4}$

d. $x = -\frac{1}{4}$, then $y = x^2 = (-\frac{1}{4})^2 = \frac{1}{16}$

How to approximate square of a number?

Rough calculation could be carried out for approximating and checking the results in squaring rational numbers. Such approximation depends on rounding off decimal numbers. Rounding is the process to estimate (approximate) particular number in context. To round a number look at the next digit in the right place, if the digit is less than 5, round down and if the digit is 5 or more than 5, round up. Rounding decimals is useful to estimate an answer easily and quickly.

Example 2.6:

Find the approximate value of x^2 in each of the following decimals

a. $x = 4.3$

b. $x = 0.026$

c. $x = 2.45$

Solution:

a) $x = 4.3 \approx 4$ [since $3 < 5$]

$x^2 = 4^2 = 16$

Therefore, $4.3^2 \approx 16$

b) $x = 0.026 \approx 0.03$

$x^2 = 0.03^2 = 0.0009$

Therefore, $0.026^2 \approx 0.0009$

c) $x = 2.45 \approx 2.5$

$x^2 = 2.5^2 = 6.25$

Therefore, $2.45^2 \approx 6.25$

Exercise 2.2.

1. Determine whether each of the following statements is true or false
   a. $0^2 = 2$
   b. $10^2 > (10.05)^2 > 11^2$
   c. $(9.9)^2 \approx 100$
2. Find the approximate value of x^2 if
   a) $x = 4.2$
   d) $x = 1.06$
   b) $x = 10.8$
   c) $x = -8.7$

2.1.2. Use of table values and scientific calculator to find squares of rational numbers

Competency: **At the end of this sub-topic, students should:**

- Calculate the square of a number

Activity: 2.2.

Discuss with your friends

1. Define scientific notation of a number by your own words.
2. Express the following numbers in scientific notation.
   a. 450
   c. 84.3
   e. 0.05
   b. 0.0045
   d. 0.256
3. Use table values of square to find x^2 for each of the following
   a. $x = 1.51$
   d. $x = 5.29$
   b. $x = 4.60$
   e. $x = 5.06$
   c. $x = 3.25$
   f. $x = 7.64$

To find the square of rational number when it is written in the form of a decimal is very tedious and time consuming work. To avoid this tedious and time consuming work a table of squares is prepared and presented in the numerical tables at the end of this book. In the table the first column headed by x lists numbers from $1.0 - 9.9$, the remaining columns are headed respectively by the digits $0\ to\ 9$.

x	0	1	2	3	4	5	6	7	8	9
1.0										
1.1										
1.2										
.										
.										
.										
9.9										

Example 2.7:

By using table of squares evaluate $(2.26)^2$

Solution:

The way how we can find the square of rational numbers from the table as follows:

step i. Find the row which start with 2.2

step ii. Move to right along the row until you get column under 4

step iii. Read the number at the intersection of the row in (ii) and column in (vi)

x	0	1	2	3	4	5	6	7	8	9
1	1	1.02	1.04	1.061	1.082	1.102	1.124	1.145	1.166	1.188
1.1	1.21	1.232	1.254	1.277	1.277	1.322	1.346	1.369	1.392	1.416
1.2	1.44	1.464	1.488	1.513	1.513	1.562	1.588	1.613	1.638	1.644
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
2.1	4.41	4.452	4.494	4.537	4.58	4.622	4.666	4.709	4.752	4.796
2.2	4.84	4.884	4.928	4.973	5.018	5.062	5.108	5.153	5.198	5.244
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Hence $(3.24)^2 = 10.50$

From the table check the value of the following square numbers are true.

a. $(3.10)^2 = 9.610$

b. $(35.2)^2 = (3.52 \times 10)^2 = (3.52)^2 \times 10^2 = 12.39 \times 100 = 1239$

c. $(0.365)^2 = (3.65)^2 \times (10^{-1})^2 = 13.32 \times 0.01 = 0.1332$

Example 2.8:

Find the square of the number 6.75 using each techniques and compare your result.

Solution:

i. Using Rough(approximate value) Calculation

$6.75 \approx 7$

$(6.75)^2 \approx (7)^2 = 49$

$(6.75)^2 \approx 49$

ii. Using Value obtain from table.

a. Find the row which start with 6.7

b. Find the column headed by 5

c. Read the number, that $(6.75)^2$ at the intersection of the row in (vii) and column in (v)

Therefore $(6.75)^2 \approx 45.56$

i. Using Scientific calculator (Exact value)

Multiply $(6.75)\ by\ (6.75)$ $(6.75) \times (6.75) = 45.5625$

Therefore $(6.75)^2 = 45.5625$

Exercise 2.3.

1. Determine whether each of the following statement is true or false

   a. $(3.22.)^2 = 10.30$ b. $(9.9)^2 = (98.01)^2$

   b. $(3.56)^2 = 30.91$

2. If $(3.67)^2 = 13.47$ then find

   a. $(36.7)^2$ b. $(367)^2$ c) $(0.367)^2$

3. If $(8.435)^2 = x$ then determine each of the following in terms of x.

    a. $(84.35)^2$ b. $(0.8435)^2$

4. Find the square of the number 8.95.

    a. Using rough calculation method
    b. Using numeral table value
    c. Using Scientific calculator

2.1.3. Square Roots of a Rational number

Competency:

At the end of this section, students should:

- Calculate the square root of perfect squares

Activity: 2.3.

Find the prime factorization of the following numbers by using the factor tree method

a. 16 c. 64 e. 400

b. 32 d. 256

Definition 2.3

For any two rational numbers a and b; if $a^2 = b$ then a is called the square root of b.

Example 2.9:

a. The square root of 4 is 2 because $2^2 = 4$
b. The square root of 9 is 3 because $3^2 = 9$
c. The square root of 16 is 4 because $4^2 = 16$
d. The square root of $\frac{25}{64}$ is $\frac{5}{8}$ because $(\frac{5}{8})^2 = \frac{25}{64}$

Note:

iii. The operation “extracting square root” is the inverse of the operation squaring.

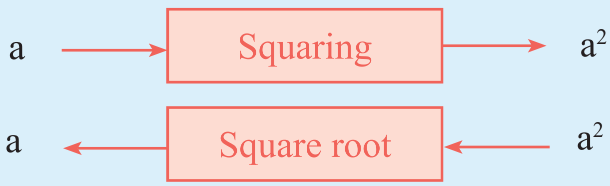
iv. In extracting square roots of rational numbers,

- first decompose the number into product consisting of two equal factors and
- take one of the equal factors as the square root of the given number.

v. The positive square root of a number is called the principal square root. The symbol "$\sqrt{\ }$" called the radical sign, is used to indicate the principal square root.

vi. For $b \geq 0$, the expression $\sqrt{b}$ is called the principal square root of b or radical b and b is called the radicand.

vii. The relation between squaring and square root can be expressed as:



viii. In power form $\sqrt{a} = a^{\frac{1}{2}}$

ix. Negative rational numbers don’t have square roots in the set of rational number.

x. The square root of zero is zero.

Example 2.10:

Find the square root of x, if x is

a. 16 c. 0.04

b. 121 d. $\frac{25}{100}$

Solution:

a. $x = 16 = 4 \times 4 = -4 \times -4$

$x = 4^2 = (-4)^2$

Thus, the square root of 16 is 4 or -4 . But $\sqrt{16} = 4$ [since "$\sqrt{\ }$" indicate the positive square root. To indicate the negative square root we use $-$ "$\sqrt{\ }$"]

b. $x = 121 = 11 \times 11 = -11 \times -11$

$x = 11^2 = (-11)^2$

Thus, the square root of 121 is 11 or -11. But $\sqrt{121} = 11$

c. $x = 0.04 = 0.2^2 = (-0.2)^2$

Thus, the square root of 0.04 is 0.2 or -0.2. But $\sqrt{0.04} = 0.2$

d. $x = \frac{25}{100} = \frac{5}{10} \times \frac{5}{10} = -\frac{5}{10} \times -\frac{5}{10}$

$x = (\frac{5}{10})^2 = (-\frac{5}{10})^2$

Thus the square root of $\frac{25}{100}$ is $\frac{5}{10}$ or $-\frac{5}{10}$. But, $\sqrt{\frac{25}{100}} = \frac{5}{10} = \frac{1}{2}$

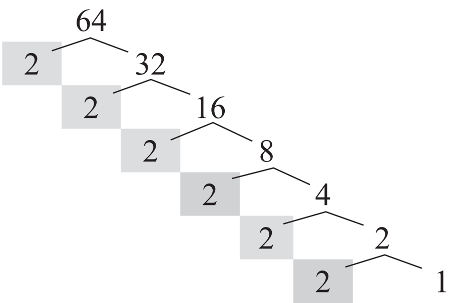
Example2.11:

Find the square root of each of the following numbers by using prime factorization.

a. 64 b. 100 c. 400

Solution:

a. 64



Now arrange the factors so that 64 is the product of two identical factors.

i.e. $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

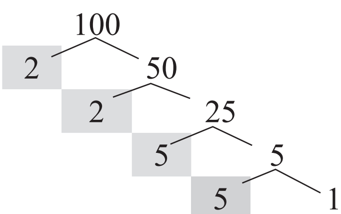
$64 = (2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$64 = 8 \times 8$

$64 = 8^2$

so $\sqrt{64} = 8$

b. 100



Arrange the factors so that 100 is the product of two identical factors

i.e.$100 = 2 \times 2 \times 5 \times 5$

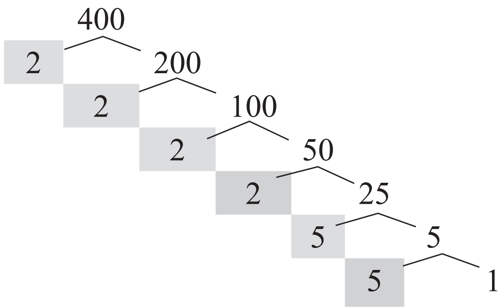
$100 = (2 \times 5) \times (2 \times 5)$

$100 = 10 \times 10$

$100 = 10^2$

So $\sqrt{100} = 10$

c. 400



Arrange the factors so that 400 is the product of two identical factors

i.e. $400 = 2 \times 2 \times 2 \times 2 \times 5 \times 5$

$400 = (2 \times 2 \times 5) \times (2 \times 2 \times 5)$

$400 = 20 \times 20$

$400 = 20^2$

So $\sqrt{400} = 20$

Exercise 2.4.

1. Evaluate the given square root.
   - *a.* $\sqrt{0}$
   - *b.* $\sqrt{169}$
   - *c.* $\sqrt{576}$
   - *d.* $\sqrt{0.25}$
   - *e.* $\sqrt{0.0016}$
2. The area of square is 144cm^2. What is the length of each side?
3. Using prime factorization technique evaluates the square root of the following numbers.

   a) 256 b) 324 c) 1225

2.1.4. Use of table values and scientific calculator to find square roots of rational numbers

Competency:

At the end of this section, students should:

- Calculate the square root of a number

Activity: 2.4.

Discuss with your friends

Use table value (Attached at the end of your text book) to evaluate the following square root.

a. $\sqrt{13.18}$ c. $\sqrt{11.16}$ e. $\sqrt{12.74}$

b. $\sqrt{98.01}$ d. $\sqrt{56.85}$

To find the square root of rational number when it is written in the form of a decimal is a tedious work. To avoid this tedious work a table of squares is prepared and presented in the numerical table at the end of this book.

Example 2.12:

Find $\sqrt{16.40}$ from the numeral table

Solution:

Step i. Find the number 16.40 on the body of the table.

Step i. On the row containing this number move to the left and read 4.0 under x.

Step i. To get the third digit start from 16.40 move vertically up ward and read 5.

Therefore $\sqrt{16.40} \approx 4.05$

Note:	If the radicand is not found in the body of the table, you can approximate to the nearest square roots of a number.

Example 2.13:

Find $\sqrt{71.80}$ from numeral table

Solution:

$\sqrt{71.80}$ is not directly in the numerical table. So find two numbers from the table to the left and to the right of 71.80.

That is, $71.74 < 71.80 < 71.91$

Find the nearest number to 71.80 from those two numbers. Which is 71.74.

Thus $\sqrt{71.80} \approx \sqrt{71.74} = 8.47$

Therefore, $\sqrt{71.80} \approx 8.47$

Exercise 2.5.

1. Find the square root of each of the following numbers from numerical table

   a. 2.310 c. 15.68 e. 95.06

   b. 4.326 d. 98.60

2. If $(4.36)^2 = 21.41$ then, find

   a. $\sqrt{21.44}$ *c.* $\sqrt{0.2144}$

   b. $\sqrt{2144}$ *d.* $\sqrt{0.003564}$

2.2. Cubes and Cube roots

2.2.1. Cube of a rational number

Competency: **At the end of this section, students should:**

- Calculate the cube root of a number

Activity: 2.5.

1. Find x^3 in each of the following rational numbers.

   a. $x = 2$ c. $x = \frac{1}{4}$ e. $x = 0$

   b. $x = -4$ d. $x = 0.5$

2. Which of those numbers are written as x^3?

   a. 8 c. 121 e. 2700

   b. 64 d. 729

Definition 2.4

A cube number is a number obtained by multiplying the number by itself three times.

Example 2.14:

The following are some cube numbers.

a. $1 \times 1 \times 1 = 1$

b. $2 \times 2 \times 2 = 8$

c. $3 \times 3 \times 3 = 27$

d. $4 \times 4 \times 4 = 64$

Example 2.15:

Find x^3 in each of the following rational numbers

a. $x = 2$

b. $x = -4$

c. $x = 0.02$

d. $x = \frac{2}{3}$

e. $x = -\frac{1}{4}$

Solution:

a. $x^3 = x \times x \times x = 2 \times 2 \times 2 = 8$

b. $x^3 = x \times x \times x = -4 \times -4 \times -4 = -64$

c. $x^3 = x \times x \times x = 0.02 \times 0.02 \times 0.02 = 0.000008$

d. $x^3 = x \times x \times x = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$

e. $x^3 = x \times x \times x = -\frac{1}{4} \times -\frac{1}{4} \times -\frac{1}{4} = -\frac{1}{64}$

Recall that: Rough calculation could be carried out for approximating and checking the results in cube of rational numbers. Such approximation depends on rounding off decimal numbers. Rounding is the process to estimating (approximating) particular number in context. To round a number, look at the next digit in the right place, if the digit is less than 5, round down and if the digit is 5 or more than 5, round up. Rounding decimals is useful to estimate an answer easily and quickly.

Example 2.16:

Find the approximate value of x^3 in each of the following decimals

a) $x = 3.2$ b) $x = 4.057$ a) $x = 10.57$

Solution:

a) $x = 3.2 \approx 3$

$x^3 = x \times x \times x \approx 3 \times 3 \times 3 = 27$

Therefore, $(3.2)^3 \approx 27$

b) $x = 4.057 \approx 4$

$x^3 = x \times x \times x \approx 4 \times 4 \times 4 = 64$

Therefore, $(4.057)^3 \approx 64$

c) $x = 10.57 \approx 11$

$x^3 = x \times x \times x \approx 11 \times 11 \times 11 = 1331$

Therefore, $(10.57)^3 \approx 1331$

Group Work 2.2

Discuss the following patterns with your groups

1 [first odd number] = 1 = 1^3,

3 + 5 [the sum of the next two odd numbers] = 8 = 2^3 ,

7 + 9 + 11 [the sum of the next three odd numbers] = 27 = 3^3 ,

13 + 15 + 17 + 19 [the sum of the next four odd numbers] = 64 = 4^3 ,

21 + 23 + 25 + + 27 + 29 [the sum of the next five odd numbers] = 125 = 5^3 ,

31 + 33 + 35 + 37 + 39 + 41 [the sum of the next six odd numbers] = 216 = 6^3 ,

43 + 45 + 47+ 49 + 51 + 53 + 55 [the sum of the next seven odd numbers] = 343 = 7^3 . What will be the sum of the next eight odd numbers?(hint: see the above pattern). How many consecutive odd numbers will be needed to obtain the sum as 100? 441?

Definition 2.5

A rational number x is called a perfect cube if and only if $x = m^3$ for some $m \epsilon \mathbb{Q}$. That is, a perfect cube is a number that is a product of three identical factors.

Example 2.17:

The numbers 1, 8, 27, 64, 125 and 216 are perfect cubes. Because

$1 = 1^3$, $27 = 3^3$ $125 = 5^3$,

$8 = 2^3$, $64 = 4^3$, $216 = 6^3$

Exercise 2.6.

1. Determine whether each of the following statement is true or false

   a. $2^3 = 2 \times 3$ c. $8^3 = 64 \times 8$ e. $(-3)^3 = -27$

   b. $-20^3 = -400$ d. $(\frac{3}{4})^3 = \frac{27}{16}$ f. $12^3 = 144 \times 12$

2. Find x^3 in each of the following

   a. $x = 4$ b. $x = 0.5$ c. $x - \frac{1}{2}$ d. $x = \frac{3}{4}$

3. Find the approximate value of x^3 in each of the following

   a. x = -3.45 b. x = 4.98 c. x = 0.025 d. x = 2.75

4. Identify whether each of the following are perfect cubes ?

   a) 42 c. 64 e. 144

   b) b) 60 d. 125 f) 216

5. What are the consecutive perfect cubes which added to obtain a sum of 100? 441?

2.2.2. Cube Root of a rational number

Competency:

At the end of this sub-topic, students should:

- Calculate the cube of a number

Activity: 2.6.

1. Define a cube root of a number by your own words.
2. Find the cube root of each of the following numbers

   a. 216 b) 729 c) 343 d) 64 e) 1331

Definition 2.6

The cube root of a given number is one of the three identical factors whose product is the given number.

Example 2.18:

a. $1 \times 1 \times 1 = 1$, so 1 is the cube root of 1

b. $2 \times 2 \times 2 = 8$, so 2 is the cube root of 8

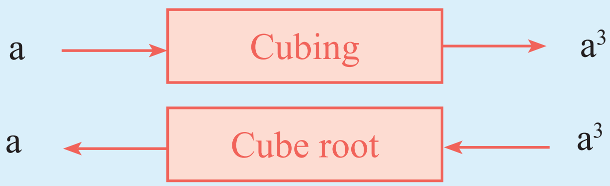
c. $4 \times 4 \times 4 = 64$, so 4 is the cube root of 64

Note:

i. 33 = 27. Then 27 is the cube of 3 and 3 is the cube root of 27. This is written as $3 = \sqrt[3]{27}$. The symbol $3 = \sqrt[3]{27}$ is read as the principal cube root of 27 or simply the cube root of 27.

ii. The symbol $\sqrt[3]{\ }$ is called a radical sign. The expression $\sqrt[3]{a}$ is called a radical, 3 is called the index and a is called the radicand. When no index is written, the radical sign indicates square root.

iii. The relation between cubing and cube root can be expressed as:



iv. $\sqrt[3]{a} = a^{\frac{1}{3}}$ [exponential form]

v. Each rational number has exactly one cube root.

Exercise 2.7.

1. State another name for $4^{\frac{1}{3}}$.
2. Write the following in exponential form

a. $\sqrt[3]{10}$ b) $\sqrt[3]{0.23}$

3. Identify whether each of the following are perfect cube.

   3, 6, 8, 9, 12, 64, 216, 729, 625, 400

4. Find the cube root of the following numbers.

   a) 216 b. -343 c. 1000 d. 1728 e. 0

2.3. Applications on squares, square roots, cubes and cube roots

Competency: **At the end of this section, students should:**

- Solve real-life problems

Activity: 2.7.

1. In figure 2.1 to the right

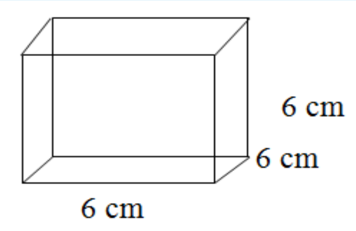


Figure2.1:

Find a) The surface area of a cube

b) The volume of a cube

c) Compare the surface area and volume of a given cube.

2. Find the area of the square with length side 5cm?

Squares, square roots, cubes and cube roots used to express many day to day activities. For instance, squares and square roots are used in all walks of life, such as carpentry, engineering, designing buildings, and technology.

Square roots are important for builders, especially when they are constructing frames and roof trusses for houses. It is important to be to work out the longest side in a rectangular frame and the diagonal in a wall.

Cube root used to solve for the dimensions of a three-dimensional object of a certain volume.

Example 2.19:

Alemu and Almaz want to make a square patio. They have concrete to make an area of 400 square meters. How long can a side of their patio be?

Solution:

Let x be the length of each sides of a square patio

$A = x^2$

$400 = x^2$ From this x can be calculated as principal square root:

$x = \sqrt{400} = 20$

Therefore, the length of each sides of a square patio is 20 m.

Exercise 2.8.

1. 225 students stand in rows in such a way that the number of rows is equal to the number of students in a row, how many students are there in each row?
2. Getaneh's flower garden is a square. If he enlarges it by increasing the width 1 m and the length 3 m, the area will be 19 square meters more than the present area. What is the length of a side now?
3. If the area of square region is 64 square meter, then what is the length sides of a square region?

UNIT SUMMARY

1. The process of multiplying a number by itself is called squaring of a number.
2. A rational number x is called a perfect square if and only if $x = m$ for some $m \in \mathbb{Q}$
3. The square of rational numbers is also rational numbers.
4. $0 \times 0 = 0$ therefore $0^2 = 0$
5. For any rational numbers a and b, $(ab)^2 = a^2b^2$
6. For any rational number, a and b, $(\frac{a}{b})^2 = \frac{a^2}{b^2}$ where $b \neq 0$
7. For any rational number x, there is a rational number y $(y \geq 0)$ such that $x^2 = y$.
8. For any two rational numbers a and b if $a^2 = b$, then a is called the square root of b, $b \geq 0$.
9. The operation "extracting square root" is the inverse of the operation squaring.
10. In extracting square roots of rational numbers ,
    - first decompose the number into product consisting of two equal factors and
    - Take one of the equal factors as the square root of the given number.
11. The positive square root of a number is called the principal square root. The symbol "$\sqrt{\ }$" called the radical sign, is used to indicate the principal square root.
12. For $b \geq 0$, the expression $\sqrt{b}$ is called the principal square root of b or radical b, b is called the radicand.
13. The relation between squaring and square root can be expressed as follow as
14. In exponent form $\sqrt{a} = a^{\frac{1}{2}}$.
15. Negative rational numbers don't have square roots in the set of rational number.
16. The square root of zero is zero.

17. A cube number is a number obtained by multiplying the number by itself three times.

18. A rational number x is called a perfect cube if and only if $x = n^3$ for some $n \in \mathbb{Q}$ That is, a perfect cube is a number that is a product of three identical factors.

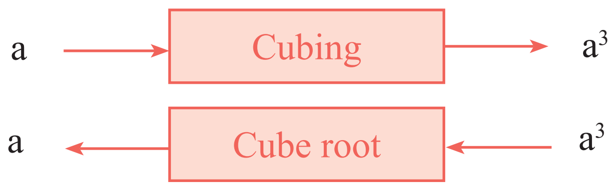
19. The cube root of a given number is one of the three identical factors whose product is the given number.

20. $4^3 = 64$, then 64 is the cube of 4 and 4 is the cube root of 64.This is written as $4 = \sqrt[3]{64}$

21. The symbol $4 = \sqrt[3]{64}$ is read as the principal cube root of 64 or simply the cube root of 64.

22. The symbol $\sqrt[3]{\ }$ is called a radical sign, the expression $\sqrt[3]{a}$ is called a radial, 3 is called the index and a is called the Radicand. When no index is written the radical sign indicates square root.

23. The relation between cubing and cube root can be expressed as:a



24. $\sqrt[3]{a} = a^{\frac{1}{3}}$ [exponential form]

25. Each rational number has exactly one cube root.

REVIEW EXERCISE

1. Determine whether each of the following statements is true or false

a. $8^2 = 8 \times 2$

b. $-2^2 = -4$

c. $(-\frac{2}{3})^2 = \frac{4}{9}$

d. $-3^3 = 27$

e. $0^3 = 3$

f. $(-5)^2 = 25$

2. Find x^2 in each of the following rational numbers

   a. $x = 0.03$ c. $x = \frac{1}{4}$

   b. $x = -2$ d. $x = -\frac{5}{4}$

3. Identify whether each of the following are perfect square.

   a. 216 c. 1000 e. 900

   b. 625 d. 729 f. 2025

4. What is the sum of the first 25 odd natural numbers?

5. Find the square of the number 8.04

   a. Using rough calculation method

   b. Using numerical table value

   c. Using Scientific calculator

6. Find the square root of x if x is

   a. 64 b. 81 c. 0.04

7. Using prime factorization technique evaluates the square root of the following numbers.

   a. 225 b. 625 c. 2500

8. Find the square root of each of the following numbers from numeral table

   a. 19.62 c. 30.03

   b. 10.56 d. 64.64

9. If $(7.43)^2 = 55.20$ then find

   a. $(74.3)^2$ b. $(743)^2$ c. $(0.743)^2$

10. If $(3.42)^2 = 11.70$, then find

   a. $\sqrt{11.70}$ b. $\sqrt{1170}$ c. $\sqrt{0.117}$

11. Identify whether each of the following are perfect cubes?

   a. 27 b. 60 c. 64 d. 25

12. Find x^3 in each of the following rational numbers.

   a. $x = 2$ c. $x = -20$

   b. $x = 0.03$ d. $x = \frac{1}{4}$

13. Identify the base , exponent, power forms and Standard numeral form for each of the following numbers

a. $2^3 = 8$ b. $3^3 = 27$ c. $10^3 = 1000$

14. Write the following in power form

a. 900 b. 324 c. 216

15. Using the following pattern

1 [first odd number] $= 1 = 1^3$

3 + 5 [the sum of the next two odd numbers] $= 8 = 2^3$

7 + 9 + 11 [the sum of the next three odd numbers] $= 27 = 3^3$

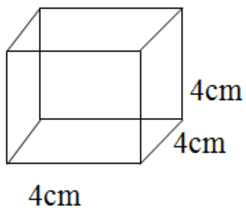
13 + 15 + 17 + 19 [the sum of the next four odd numbers] $= 64 = 4^3$

21 + 23 + 25 + 27 + 29 [the sum of the next five odd numbers] $= 125 = 5^3$

What will be the sum of six consecutive odd numbers next to 29?

16. In figure 1.1 as shown find:

a. The surface area of a cube



b. The volume of a cube

c. Compare the surface area and volume of a given cube.

17. Find the area of the square with length sides 10cm

Unit 3

3. LINEAR EQUATIONS AND INEQUALITIES

Learning Outcomes:

At the end of this unit, learners will able to:

- Graph linear equations of the type y = mx + c
- Solve linear inequalities.
- Solve application of linear inequalities.

Main content

INTRODUCTION

In grade 7, you have learnt about linear equations in one variable and the method to solve them, and graphs of $y = mx, m \in \mathbb{Q}, m \neq 0$. In this unit, we will discuss about linear inequalities and the methods to solve them and we sketch graphs of the form $y = mx + c, m \in \mathbb{Q}, m \neq 0$. Finally, we apply the knowledge about linear equations and inequalities to solve word problems.

3.1. Revision of Cartesian coordinate system

Competency: **At the end of this sub-unit, students should:**

- Describe the Cartesian coordinate system.

Activity: 3.1.

1. Describe the following terms in your own words.
   a. Cartesian coordinate plane
   b. Ordered pair
   c. Quadrant
2. Draw a pair of coordinate axes, and plot the points associated with each of the following ordered pair of numbers.
   a. (0, 0) b. (-2, 1) c. (4, -3) d. (0, 5)
3. In which quadrant are the following points located?
   a. (7, -2) b. (-3, 5) c. (-9, -11)
4. Name the quadrant in which the point P(x, y) lies when:
   a. $x > 0 , y < 0$ c. $x < 0 , y > 0$
   b. $x < 0, y < 0$ d. $x > 0 , y > 0$
5. Consider the equation $y = 3x - 1$.
   a. Determine the values of y when the values of x are -1, 0, 1
   b. Plot the ordered pairs on the Cartesian coordinate plane.

To determine the position of a point in a Cartesian coordinate plane, you have to draw two intersecting perpendicular number lines. The two intersecting perpendicular lines are called axes, the horizontal line is the x – axis and the vertical line is the y – axis. Usually the arrows indicate the positive direction.These axes intersect at a point called the origin. These two axes together form a plane called the Cartesian coordinate plane. The position of any point in the plane can be represented by an ordered pair of numbers (x, y). These ordered pairs are called the coordinates of the point. The first coordinate is called the x- coordinate or abscissa and the second coordinate is called the y- coordinate or ordinate. The two axes divide the given plane into four quadrants. Starting from the positive direction of the x-axis and moving the anticlockwise (counter clock wise) direction, the quadrants which you come across are called the I, the II, the III, and the IV quadrants respectively.

Example 3.1:

The point with coordinates (2, 5) has been plotted on the Cartesian plane as follow. Imagine a vertical line through 2 on the x-axis and a horizontal line through 5 on the y-axis. The intersection of these two lines is the point (2, 5). This point is 2 units to the right of the y-axis and 5 units up from the x-axis.

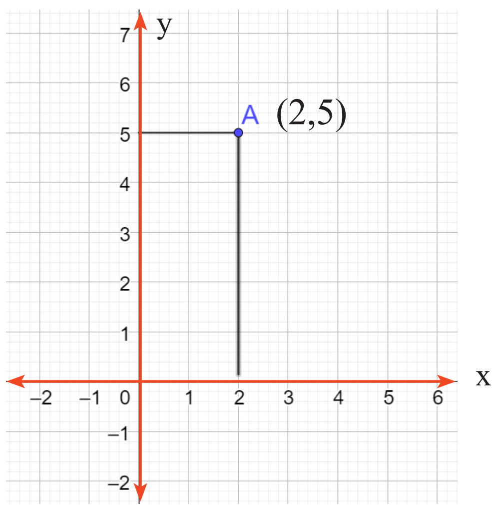


Figure3.1:

Note:

In the I quadrant, all points are (+, +)

In the II quadrant, all points are (-, +)

In the III quadrant, all points are (-, -)

In the IV quadrant, all points are (+, -)

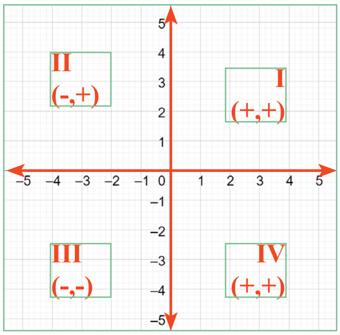


Figure3.2:

Example 3.2:

In which quadrant the following points lie?

a. $(2, -6)$ b. $(-3, 3)$ c. $(5, 0)$

Solution:

a. Since $x = 2 > 0$ and $y = -6 < 0$ then the point $(2, -6)$ lies in quadrant IV.

b. Since $x = -3 < 0$ and $y = 6 > 0$ then the point $(-3, 3)$ lies in quadrant II.

c. The point (5, 0) lies on the positive x- axis. It is neither of any quadrants.

Exercise 3.1.

1. Determine the quadrant(s) in which (x, y) is located so that the condition(s) is (are) satisfied?

   a. $x > 0$ and $y < 0$
   b. $x = -4$ and $y > 0$
   c. $y < -5$
   d. $x < 0$ and $-y > 0$
   e. $x > 2$ and $y = 3$
   f. $x > 0$

2. Find the coordinates of the point.

   a. The point is located 5 units to the left of the y-axis and 2 units above the x-axis.
   b. The point is located on the x-axis and 10 units to the left of the y-axis.

3.2. Graphs of Linear Equations

Competency: **By the completion of this section, students should:**

- Draw linear equations like $y = mx + c$ in a Cartesian coordinate plane

In this section, we will explore some basic principles for graphing.

Activity: 3.2.

1. Draw the graphs of the following lines.
   a. $x = 3$ b. $y = 3$
2. write true if the statement is correct and false if it is incorrect
   a. The line $x = 5$ is a vertical line.
   b. The line $y = -5$ is a horizontal line.
   c. The line $2x - 7 = 3$ is a horizontal line.
   d. The line $6 + 3y = -12$ is a vertical line.
3. Write an equation representing
   a. The x- axis
   b. The y- axis
4. Evaluate: $5x - 2$ when $x = -3$, $x = 0$, and $x = 2$
5. Solve for y if $3x + 5y = 4$.
6. Let $x = b$. Where does the graph lies on the coordinate plane
   a. If $b < 0$?   b. If $b = 0$?   c. If $b > 0$?
7. Let $y = b$. Which coordinate is constant, x-coordinate or y- coordinate? Which coordinate varies, x-coordinate or y- coordinate?

Revision on vertical and horizontal lines

In a vertical line all points have the same x- coordinate, but the y-coordinate can take any value. The equation of the vertical line through the point $P(a, b)$ is $x = a$. This line is parallel to the y-axis and perpendicular to the x-axis. Similarly, in a horizontal line all points have the same y – coordinate but the x- coordinate can take any value. The equation of the horizontal line through the point $P(a, b)$ is $y = b$. This line is parallel to the x-axis and perpendicular to the y-axis.

Example 3.3:

Draw the graphs of

a. x = 5 b. y = −4

Solution:

a. First construct tables of values for x and y in which x is constant.

x	5	5	5	5	5	5	5	5	5
y	−4	−3	−2	−1	0	1	2	3	4

Then, plot these points on the Cartesian coordinate plane and join them. What do you realize? All the points lie on the vertical line.

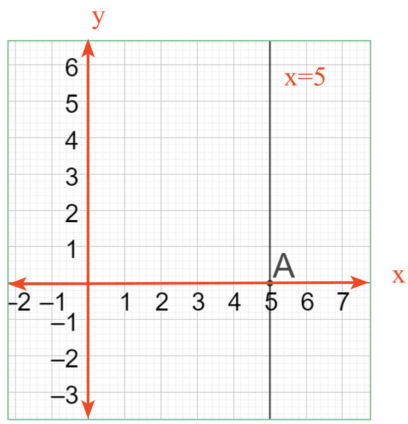


Figure3.3:

b. First construct tables of values for x and y in which y is constant.

x	−5	−4	−3	−2	−1	0	1	2	3
y	−4	−4	−4	−4	−4	−4	−4	−4	−4

Then plot these points on the Cartesian coordinate plane and join them. What do you realize? All points lie on the horizontal line.

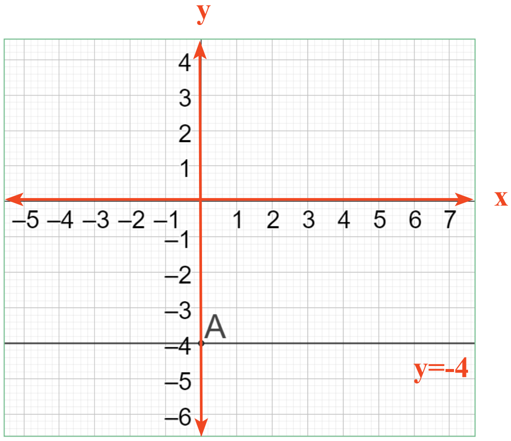


Figure3.4:

Graph of an equation of the form $y = mx$ ($m \in \mathbb{Q}, m \neq 0$)

There are several methods that can be used to graph a linear equation. The method we used at the start of this section to graph is called plotting points, or the point – plotting method.

Activity: 3.3.

1. Define a linear equation

   a. in one variable.
   b. in two variables.

2. Which of the following is a linear equation in one variable?

   a) $3x + 1 = 4 - x$
   b) $x + 4 = \frac{1}{2}(2x + 3)$
   c. $3(2x - 4) = 2(3x - 6)$

3. Refer question number 2,

   a. How many solutions do you get for each equation?
   b. What can you conclude about number of solutions for a linear equation in one variable?

4. Which of the following are linear equations in two variables?

   a. $x = -5$
   b. $x = 7 - 2y$
   c. $y = \frac{1}{2}$
   d. $3x + y = 4$

5. Given the equation y = 4x.

   a. Are the points (-1, 4), (0, 0), (2, 8) and (3, 9) satisfy the equation?

   b. When you plot points on the Cartesian coordinate plane, Which points lie on the same line?

   c. Try to generalize about points on the line and solutions of the equation.

6. For which equation, does the line pass through the origin?

   a. y = 3 c. y = -4x

   b. x = 2 d. y = 3x

From the above activity, question number 5, we observe that for each value of xx, there is one corresponding value of y. This relation is represented by an ordered pair (x, y). The set of all those ordered pairs that satisfy the equation $y = 4x$ is the solution of the equation $y = 4x$.

Graph of an equation in xx and y is the set of all points (x, y) in the coordinate plane that satisfy the equation.

Definition 3.1

[Graph of a Linear Equation $y = mx$ $(m \in \mathbb{Q}, m \neq 0)$]

The graph of a linear equation y = mx is a straight line passes through the origin.

Note:	
	✶ Every point on the line is a solution of the equation of a line.
	✶ Every solution of the equation is a point on the line.

Example 3.4:

Sketch the graphs of the following equations on the same Cartesian coordinate plane

c. y = 3x d. $y = -3x$

Solution:

To sketch the graphs of the equations follow the following steps.

Step 1: Choose some values for x.

Let x = −2, −1, 0, 1, and 2

Step 2: Put these values of x into the equation to get the values of y.

a. y =3x

when x = -2, y = 3(-2) = -6

when x = - 1, y = 3(-1) = -3

when x = 0, y = 3(0) = 0

when x = 1, y = 3(1) = 3

when x = 2, y = 3(2) = 6

Step 3: Write these pairs of values in a table.

x	−2	−1	0	1	2
y	−6	−3	0	3	6
(x,y)	(−2,−6)	(−1,−3)	(0,0)	(1, 3)	(2,6)

Step 4: Plot the points on the Cartesian plane and join them.

Step 5: Label the line y = mx.

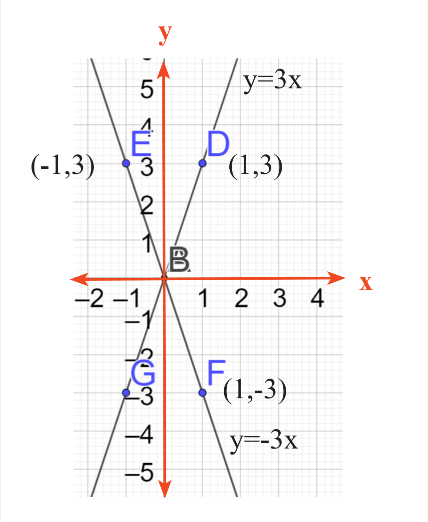


Figure3.5:

b. y = −3x

when x = −2, y = −3(−2) = 6

when x = −1, y = −3(−1) = 3

when x = 0, y = −3(0) = 0

when x = 1, y = −3(1) = −3

when x = 2, y = −3(2) = −6

x	-2	-1	0	1	2
y	6	3	0	-3	-6
(x, y)	$(-2, 6)$	$(-1,3)$	$(0,0)$	$(1,-3)$	$(2,-6)$

If the point (k, 5) lies on the line $y = -3x$, then what is the value of k?

Since, the point is on the line, the point satisfy the equation of the line.

That is, $5 = -3(k)$

$$k = -\frac{5}{3}$$

Note:

- ✶ All ordered pairs that satisfy each linear equation of the form
- ✶ $y = mx \ (m \in \mathbb{Q}, m \neq 0)$ lies on a straight line that passes through the origin.
- ✶ The graph of the line $y = mx \ (m \in \mathbb{Q}, m \neq 0)$ passes through the I and III quadrants if $m > 0$, and the graph passes through the II and IV quadrants if $m < 0$.
- ✶ In order to draw a straight line, you need to find any two points or coordinates through which the line passes.

Exercise 3.2.

1. Complete the following tables for sketching the graph of

a. $y = -5x$ c. $y = x$

b. $y = \frac{1}{2}x$ d. $y = -x$

x	-1	0	1	3
y				
(x, y)				

2. Sketch the graphs of the following equations on the same Cartesian coordinate plane.

   a. $y + 7x = 0$

   b. $3y = 6x$

   c. $\frac{1}{4}x - \frac{y}{2} = 0$

3. If the point P(a, 3) lies on the line given by the equation 12x-2y=0 then find the value of a.

4. Answer the following questions below by referring the figure3.6.

   a) If x = 3, what is the value of y?

   b) If y = -4, what is the value of x?

   c) Can you find the point P(0, 0) exactly on the line?

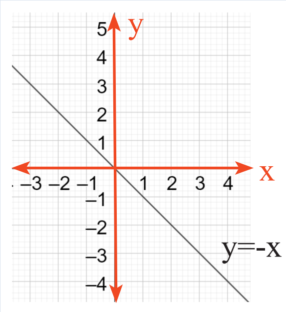


Figure3.6:

Graph of an equation of the form $y = mx + c\ (m \in \mathbb{Q}, m \neq 0)$

Example 3.5:

5. Given that $y = 3x - 5$. Decide whether the ordered pairs given below are a solution to the equation?

   a. $(0, -5)$ b. $(3, 4)$ c. $(-2, -11)$ d. $(-1, -2)$

Solution:

Substitute the x- and y- values into the equation to check whether the ordered pair is a solution to the equation.

b. $(0, -5)$

$y = 3x - 5.$

$-5 \ ?\ 3(0) - 5$

$-5 = -5$

$(0, -5)$ is a solution.

b. $(3, 4)$

$y = 3x - 5.$

$4 \ ?\ 3(3) - 5$

$4 = 4$

$(3, 4)$ is a solution.

c. $(-2, -11)$

$y = 3x - 5$

$-11 \; ? \; 3(-2) - 5$

$-11 = -11$

$(-2, -11)$ is a solution.

d. $(-1, -2)$

$y = 3x - 5$

$-2 \; ? \; 3(-1) - 5$

$-2 \neq -8$

$(-1, -2)$ is not a solution.

Example 3.6:

a. Sketch the graph of the equation y = 2x + 1 by plotting points.

Solution:

Choose any value for x and solve for y.

Let x = -1

y= 2x +1

y= 2(-1) + 1

y= -1

Let x = 0

y= 2x + 1

y = 2(0) + 1

y = 1

Let x = 1

y= 2x + 1

y = 2(1) + 1

y = 3

Then, organize the solutions in a table

x	−1	0	1
y	−1	1	3
(x, y)	(−1, −1)	(0, 1)	(1,3)

Now, we plot the points on the Cartesian coordinate plane and draw the line through these points.

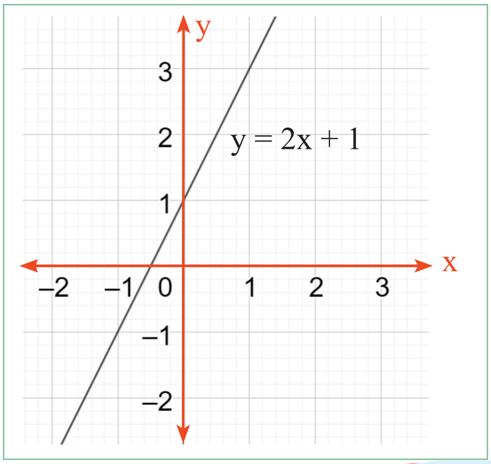


Figure3.7:

b. Sketch the graph of the equation $y = -2xy = -2x + 1$ by plotting points.

Solution:

Choose any value for x and solve for y.

Let $x = -1$	Let $x = 0$	$x = 1$
$y = -2x + 1$	$y = -2x + 1$	$y = -2x + 1$
$y = -2(-1) + 1$	$y = -2(0) + 1$	$y = -2(1) + 1$
$y = 3$	$y = 1$	$y = -1$

Then, organize the solutions in a table

x	-1	0	1
y	3	1	-1
(x, y)	$(-1, 3)$	$(0, 1)$	$(1, -1)$

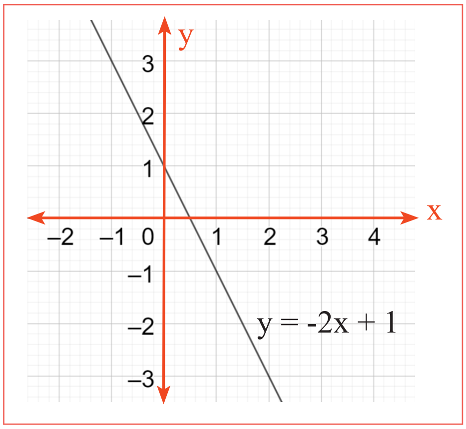


Figure3.8:

c. Sketch the graph of the equation $3y - 2x = 1$ by plotting points.

Solution:

Choose any value for x and solve for y.

Let $x = -1$ Let $x = 0$ Let $x = 1$

$3y - 2x = 1$ $\quad$ $3y - 2x = 1$ $\quad$ $3y - 2x = 1$

$3y - 2(-1) = 1$ $\quad$ $3y - 2(0) = 1$ $\quad$ $3y - 2(1) = 1$

$y = -\frac{1}{3}$ $\quad$ $y = \frac{1}{3}$ $\quad$ $y = 1$

Then, organize the solutions in a table

x	-1	0	1
y	3	1	-1
(x, y)	$(-1, -\frac{1}{3})$	$(0, \frac{1}{3})$	$(1,1)$

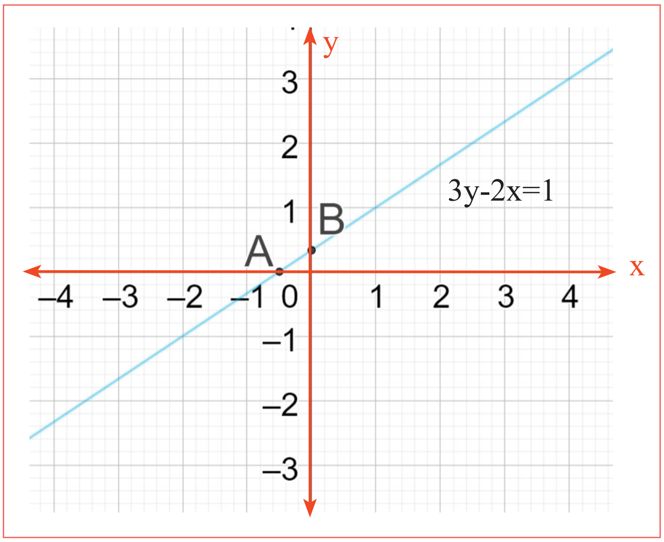


Figure3.9:

Exercise 3.3.

Sketch the graph of the following equations.

a. $y = x+3$

b. $2y - x = 6$

c. $y = x - 3$

d. $3y = x + 5$

3.3. Solving Linear Inequalities

Competency:

At the end of this sub-section, students should:

- Solve Linear Inequalities

In grade 7 mathematics lessons, you have learnt about linear equations. Now in this subtopic, you will learn more about linear inequalities.

Activity: 3.4.

1. Solve each of the following linear equations by the rules of transformation.
   a. $3(2x - 6) = 5x$
   b. $6x - 13 = 17 - (x - 2)$
   c. $0.5x + 0.5 = 0.2x + 2$
   d. $(3x - 1) - 4(2x + 1) = 4(4x - 3)$
2. Which of the following are linear inequalities?
   a. $\frac{3}{2}x < 6x + 3$
   b. $2x \geq x + 2^2$
   c. $x - 1 \leq \frac{1}{2}x - 5$
   d. $7x + \frac{1}{2}x = 4$
3. Solve each of the following linear inequalities in the set of $\mathbb{W}$, $\mathbb{Z}$, $\mathbb{Q}$.
   a. $10x < 23$
   b. $-2x > 5$
   c. $\frac{1}{2}x > 4$
   d. $\frac{2}{3}x < 6$

Definition 3.2

A mathematical sentence which contains one of the relation signs: $<$, $>$, $\leq$ $\geq$ or $\neq$ are called inequalities.

Example 3.7:

Some examples of inequalities are:

a. $2x > 0$

b. $\frac{3}{5}x + 20 \leq 0$

c. $x - 1 \neq \frac{1}{2}x - 5$

d. $2x^3 - 1 < \frac{1}{2}x + 15$

Definition 3.3

A linear inequality in one variable "x" is an inequality that can be written in the form of $ax + b < 0$, $ax + b \leq 0$, $ax + b > 0$ or, $ax + b \geq 0$ where a and b are rational numbers and $a \neq 0$.

Example 3.8:

Some examples of linear inequalities are:

a. $x + 8 > 3$

b. $x - 0.35 \leq 0.25$

c. $4 - 3x \geq 1 + \frac{3}{2}x$

Solutions of Linear Inequalities

Consider the inequality $3x - 10 \leq 0$. If we substitute the variable x by a number, we will get a true or false mathematical statement.

For instance,

x	−3	−2	−1	0	1	2	3	4
$3x - 10$	−19	−16	−13	−10	−7	−4	−1	2

From the above table, we observe that $3x -- 10 \leq\leq 0$ is true for $x < 4$. This systematic trial (testing) method is tedious and also it cannot be always used to solve all types of problems. There is a more systematic method of solving inequalities. This method follows a procedure similar to the one we use for solving equations. This method depends on equivalent transformations.

Solving an inequality means applying the appropriate transformation rules to the given inequality get a value for the variable. The numbers that replace the variable and satisfy the inequality is called the solution of the inequality.

Rules of Transformation for Inequalities

There are rules that can help to transform a given true inequality to an equivalent inequality and finally at the solution set of the inequalities.

Definition 3.4

Any two inequalities with the same solution set are called equivalent inequalities.

Example 3.9:

$5x + 6 > 2x$ and $x > -2$ are equivalent inequalities.

The following rules are used to transform a given inequality to an equivalent inequality.

Rule 1: If the same number is added to or subtracted from both sides of an inequality, the direction of the inequality is unchanged.

That is for any rational numbers a,b and c.

i. If $a < b$, then $a + c < b+c$.

ii. If $a < b$, then $a - c < b - c$.

Rule 2: If both sides of an inequality are multiplied or divide by the same positive number, the direction of the inequality is unchanged.

That is for any rational numbers a, b and c.

i. If $a < b$ and $c > 0$, then $ac < bc$.

ii. If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$ (provided $c \neq 0$).

Rule 3: If both sides of an inequality are multiplied or divided by the same negative number, the direction of the inequality is reversed.

That is for any rational numbers a, b and c.

i. If $a < b$ and $c < 0$, then $ac > bc$.

ii. If $a < b$ and $c < 0$, then, $\frac{a}{c} > \frac{b}{c}$ (provided $c \neq 0$).

Example 3.10:

Which of the following pairs of inequalities are equivalent inequalities in $\mathbb{Q}$?

a. $x + \frac{1}{2} < \frac{5}{2}$ and $x + 1 < 3$

b. $\frac{1}{2}x - 2x > 32$ and $x < -\frac{1}{2}$

Solution:

a. $x + \frac{1}{2} < \frac{5}{2}$

$= x + \frac{1}{2} - \frac{1}{2} < \frac{5}{2} - \frac{1}{2}$ [Subtracting the same number from both sides]

$= x < 2$ and

$x + 1 < 3$

$x + 1 - 1 < 3 - 1$ [Subtracting the same number from both sides]

$x < 2$

Since they have the same solution, $x + \frac{1}{2} < \frac{5}{2}$ and $x + 1 < 3$ are equivalent inequalities.

b. $\frac{1}{2}x - 2x > 32$

$= 2(\frac{1}{2}x - 2x) > 2 \times 32$

$= x - 4x > 64$

$= -3x > 64$

$= -\frac{1}{3}(-3x) < -\frac{1}{3} \times 64$

$x < -\frac{64}{3}$

Therefore, $\frac{1}{2}x - 2x > 32$ and $x < -\frac{1}{2}$ are not equivalent inequalities.

Exercise 3.4.

1. Insert the correct sign ($<$, $>$, $\leq$ $\geq$ or $\neq$) in the given blank.

   a. If $x - 5 > 2x$ x – 5 > 2x, then x _____ -5

   b. If $\frac{1}{2}x + 2 \leq 3(x-1)$ then $\frac{5}{2}x$ _____ 5

   c. If -3(x + 1) > 1, then x + 3 _____ $\frac{5}{3}$

2. Which of the following pairs of inequalities are equivalent?

   a. $3x-2<x+1$ *and* $2x+1<4$

   b. $6(2-x)<12$ *and* $3(2-x)<6$

   c. $\frac{x}{4}-1\geq\frac{3}{2}-x$ *and* $x+1>3$

   d. $5x-\frac{3}{7}\neq\frac{4}{7}-3x$ *and* $x\neq\frac{1}{8}$

Solutions of Linear inequalities by means of equivalent transformations

The objective of this subtitle is to use the rules of transformation in solving linear inequalities by getting successive equivalent inequalities until we arrive at an inequality of the form $x < a$ or $x > a$ [or $x \leq a$ or $x \geq a$]

First, we will deal with simple inequalities which we need only one equivalent transformation to obtain the form $x < a$ or $x > a$ [or $x \leq a$ or $x \geq a$]

Example 3.11:

Solve each of the following inequality in the domain of $\mathbb{Q}$.

a. $x + 2.4 \leq 6.4$ b. b) $\frac{1}{2}x > \frac{5}{2}$

Solution:

a. $x + 2.4 \leq 6.4$

$x + 2.4 - 2.4 \leq 6.4 - 2.4$ [subtracting 2.4 from both sides]

$x \leq 4, x \in \mathbb{Q}$

b. $\frac{1}{2}x > \frac{5}{2}$

$2 \times \frac{1}{2}x > 2 \times \frac{5}{2}$ [Multiplying both sides by 2]

$x > 5, x \in \mathbb{Q}$

Next we will see inequalities which require more than one equivalent transformation.

Example 3.12:

1. Solve each of the following inequality in the domain of $\mathbb{Q}$.

a. $3x - 7 \geq 11$

b. $\frac{1}{4}x + \frac{3}{2} < \frac{1}{3}$

c. $x + 4 \leq 4x - 3$

d. $-7(x + 1) < 4(x - 3) + 6$

e. $\frac{1}{2}x - 4 < \frac{1}{4}(2x - 5)$

Solution:

a. $3x - 7 \geq 11$

$3x - 7 + 7 \geq 11 + 7$

$3x \geq 18$

$\frac{1}{3} \times 3x \geq \frac{1}{3} \times 18$

$x \geq 6, x \in \mathbb{Q}$

b. $\frac{1}{4}x + \frac{3}{2} < \frac{1}{3}$

$\frac{1}{4}x + \frac{3}{2} - \frac{3}{2} < \frac{1}{3} - \frac{3}{2}$

$\frac{1}{4}x < -\frac{7}{6}$

$4 \times \frac{1}{4}x < 4 \times -\frac{7}{6}$

$x < -\frac{14}{3}, x \in \mathbb{Q}$

c) $x + 4 \leq\leq 4x - 3$

$x + 4 - 4 \leq 4x - 3 - 4$

$x \leq 4x - 7$

$x + 7 \leq 4x - 7 + 7$

$x + 7 \leq 4x$

$x + 7 - x \leq 4x - x$

$7 \leq 3x$

$\frac{7}{3} \leq \frac{3x}{3}$

$\frac{7}{3} \leq x$

$x \geq \frac{7}{3}, x \in \mathbb{Q}$

d) $-7(x + 1) < 4(x - 3) + 6$

$-7x - 7 < 4x - 12 + 6$

$-7x - 7 + 7 < 4x - 6 + 7$

$-7x < 4x + 1$

$-7x - 4x < 4x + 1 - 4x$

$-11x < 1$

$\frac{-11}{-11}x > \frac{1}{-11}$

$x > -\frac{1}{11}, x \in \mathbb{Q}$

e. $\frac{1}{2}x - 4 < \frac{1}{4}(2x - 5)$

$\frac{1}{2}x - 4 < \frac{1}{2}x - \frac{5}{4}$

$$\frac{1}{2}x - 4 + 4 < \frac{1}{2}x - \frac{5}{4} + 4$$

$$\frac{1}{2}x < \frac{1}{2}x + \frac{11}{4}$$

$$\frac{1}{2}x - \frac{1}{2}x < \frac{1}{2}x + \frac{11}{4} - \frac{1}{2}x$$

$0 < \frac{11}{4}$ which is always true.

Therefore, any rational numbers satisfy the given inequality.

Note:	
	✶ If an inequality, by means of equivalent transformations, leads to an equivalent inequality which is a true statement, then every element of the domain is a solution of the inequality. ✶ If an inequality, by means of equivalent transformations, leads to an equivalent inequality which is a false statement, then no element of the domain is a solution of the inequality.

For instance: $5x + \frac{3}{4} > 11x + 1 - 6x$ has no solution in the set of rational numbers.

1. Solve the inequality $3x < 9x + 4$ and sketch the solution set on the number line.

Solution:

$$3x < 9x + 4$$

$$3x - 9x < 9x + 4 - 9x$$

$$-6x < 4$$

$$-\frac{1}{6}(-6x) > -\frac{1}{6}(4)$$

$$x > -\frac{2}{3}$$

The solution set consists of all numbers greater than $-\frac{2}{3}$. These numbers are graphed as:

$-\frac{2}{3}$

Figure3.10:

Exercise 3.5.

1. Solve each of the following inequality in the domain of $\mathbb{Q}$
   a. $2y - 3 < \frac{1}{2}(7-y)$
   b. $(x - 2) + 4(2x + 1) \geq 4(x-3)$
   c. $\frac{1}{2}(3x-\frac{2}{3}) + 6 < x+5$
2. Is there any value of x in $\mathbb{Q}$ with the property that:
   a. $2x < 2x - 3$?
   b. $8x > 2(4x - 3)$?

3.4. Applications of Linear Equations and Inequalities

Competencies:

At the end of this section, students should:

- Apply linear equations and inequalities in real life situation.
- Solve linear equations and inequalities real- life problems.

In this sub-topic, you will apply the knowledge acquired on equations and inequalities. Different word problems that relate to our life can be solved using linear equations and inequalities. In order to solve word problems involving linear equations or inequalities, we need to translate verbal sentences into mathematical statements. That is, to solve word problems involving linear equations or inequalities:

i. Carefully read the problem and assign a variable to the unknown
ii. Interpret the word problem with mathematical statement
iii. Finally, solve the unknown

I) Applications of Linear Equations

Example 3.13:

Translate the following algebraic expressions in different word phrases.

a. $x - 5$ b. $8x$

Solution:

x-5	8x
✶ A number minus five	✶ A number multiplied by eight
✶ The difference of a number and five	✶ The product of a number and eight
✶ Five subtracted from a number	✶ Eight times a number
✶ A number decreased by five	
✶ Five less than a number	

Example 3.14:

The relationship between the temperature readings in Celsius scale C and Fahrenheit scale F is given by $C = \frac{5}{9}(F - 32)$.

a. Express F in terms of C.

b. Using the above relation of C and F, What interval on the Celsius scale corresponds to the temperature of $50 < F$?

Solution:

a. $C = \frac{5}{9}(F - 32).$

$9C = 9 \times \frac{5}{9}(F - 32).$

$9C = 5(F - 32).$

$\frac{9}{5}C = F - 32$

$F = \frac{9}{5}C + 32$

b. $F = \frac{9}{5}C + 32\frac{9}{5}C + 32$

Since $F > 50$, $F = \frac{9}{5}C + 32 > 50$

$\frac{9}{5}C > 50 - 32$

$C > 10$

Example 3.15:

Three tractor drivers working on a private farm together ploughed 12.4 hectares of land in a shift. The first driver ploughed half of the second and the second ploughed 2.4 hectares more than the third. Find the amount of hectare ploughed by the second driver.

Solution:

Total land ploughed = 12.4 hectares

Land ploughed by the 1st driver = half of the 2nd

Land ploughed by the 2nd driver = 2.4 hectare more than the 3rd

Let the land ploughed by the 3rd driver be x hectare.

Then,

$$x + (x + 2.4) + \frac{1}{2}(x + 2.4) = 12.4$$

$$2x + 2.4 + \frac{1}{2}x + 1.2 = 12.4$$

$$\frac{5}{2}x + 3.6 = 12.4$$

$$\frac{5}{2}x = 8.8$$

$$x = 3.52$$

Therefore, the second driver ploughed 3.52 + 2.4 = 5.92 hectares.

Example 3.16:

Three years ago the sum of the ages of a man and his son was 52 years. Now the man is 18 years older than his son. What is the present age of his son?

Solution:

Let M = the man's present age and S = the son's present age.

Then, $(M-3) + (S-3) = 52$. But $M = S + 18$

$$(M-3) + (S-3) = 52$$

$$M + S - 6 = 52$$

$$M + S = 58$$

$$S + 18 + S = 58$$

$$2S = 40$$

$$S = 20$$

Therefore, the present age of his son is 20 years.

Example 3.17:

Samuel can do a certain work in 15 days and Saron can do the same work in 10 days. In how many days do they together to finish the work?

Solution:

Let Samuel and Saron together can finish the work in d days.

Let us make the following table

Name	Number of days for doing the work	Part of the work done in one day	Work done in d days.
Samuel	15	$\frac{1}{15}$	$\frac{d}{15}$
Saron	10	$\frac{1}{10}$	$\frac{d}{10}$

By the equation, $\frac{d}{15} + \frac{d}{10} = 1$

$$d\left(\frac{1}{15} + \frac{1}{10}\right) = 1$$

$$d\left(\frac{2+3}{30}\right) = 1$$

$$d = 6$$

Therefore, they together can finish the work in 6 days.

Exercise 3.6.

1. Write the following sentences in a mathematical symbol.
   a. 5 less than 7 times a number is 0.
   b. The quotient of a number and nine is 2 less than the number.
   c. Multiply a number by 2 and add 4, the result you get will be 3 times the number decreased by 7
2. Solve for the variable M in the equation $F = G\frac{mM}{r^2}$.

3. The sum of the ages of the mother and her daughter is 68 years. The mother is 22 years older than her daughter. How old is her mother?
4. In a class there are 42 students. The number of girls is 1.1 times the number of boys. How many boys and girls are there in the class?

II) Application of Linear Inequalities

Example 3.18:

Translate the following expressions involving inequalities

a. The cost p of a pen is less than two times the cost b of a book.
b. The speed of a car, v, was at least 30 km/h.
c. Multiply a number a by two so that it is not less than three times a number b.
d. Adding 7 to three times a number x exceeds 29

Solution:

a. $p < 2b$
b. $v \geq 30$
c. $2a \geq 3b$
d. $3x + 7 > 29$

Note:	The following mathematical interpretations can be used for the phrase: ⅄ x is at least a ________ $x \geq a$ ⅄ x is not less than a ________ $x \geq a$ ⅄ x is at most a ________ $x \leq a$ ⅄ x is not more than a ________ $x \leq a$

Example 3.19:

Find two consecutive even positive integers whose sum is at most 10.

Solution:

Let x be the first even integer. Then the second will be x + 2.

$$x + (x + 2) \leq 10$$

$$2x + 2 \leq 10$$
$$2x \leq 8$$
$$x \leq 4$$

Since x is a positive even integer, x = 2 or x = 4.

Therefore, the numbers are 2 and 4, or 4 and 6.

Example 3.20:

Three years ago a father's age exceeded at least four times his son's age. If the father is 47 years old now, then what is the possible present age of the son?

Solution:

Let the son's age be S and father's age be F. Then

$$F - 3 > 4(S - 3)$$
$$F - 3 > 4S - 12$$
$$F > 4S - 9$$
$$47 > 4S - 9$$
$$56 > 4S$$
$$S < 14$$

Example 3.21:

Hanan got a new job and will have to move. Her monthly income will be Birr 13,926. To qualify to rent an apartment; Hanan's monthly income must be at least three times as much as the rent. What is the highest rent Hanan will qualify for?

Solution:

Let x be the amount of rent.

Hanan's monthly income must be at least three times the rent. So it can be written in a linear inequality as $13{,}926 \geq 3x$

$$3x \leq 13{,}926$$

$$\frac{3x}{3} \leq \frac{13{,}926}{3} \qquad x \leq 4642$$

Therefore, the highest rent amount is Birr 4642.

Example 3.22:

A carnival has two plans for tickets.

Plan A: Birr 15 for entrance fee and Birr 10 for each ride.

Plan B: Birr 10 for entrance fee and Birr 15 for each ride. How many rides would you have to take for plan A to be less expensive than plan B?

Solution:

Let x be the number of rides.

Then cost with plan A = $15 + 10x$

cost with plan B = $10 + 15x$

since, cost with plan A < cost with plan B

$15 + 10x < 10 + 15x$

$15 - 10 < 15x - 10x$

$5 < 5x$

$1 < x$

So if you plan to take more than one ride, plan A is less expensive.

Exercise 3.7.

1. Translate the following expressions involving inequalities.
   a. Three times a number decreased by four is at most twenty.
   b. The ratio of a number to seven is less than ten.
   c. The height of a roof, h, was no more than 6m.
   d. The sum of two consecutive integers is smaller than three times the smaller integer.
2. Five times a certain natural number is decreased by two times the number is less than 12. What are the possible values of this number?
3. Imagine that you are taking a ride while on vacation. If the ride service charges Birr 50 to pick you up from the hotel and Birr 10 per km for the trip. What minimum km's you travel if your cost is not more than Birr 1350.

UNIT SUMMARY

1. Two mutually perpendicular lines, called axes, form the Cartesian coordinate plane.
2. The two axes divide the given plane into four quadrants called I, II, III, and IV quadrants respectively starting from the positive direction of the x- axis and moving to the counter clock wise.

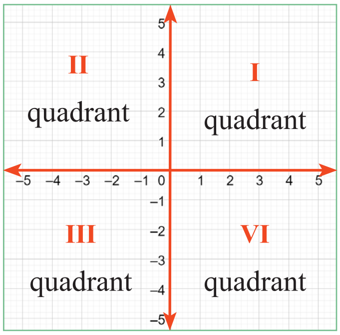


Figure3.11:

3. All ordered pairs that satisfy a linear equation $y = mx$, m $\in \mathbb{Q}, m \neq 0$ is a straight line that passes through the origin. If $m > 0$, the line passes through the I and III quadrant, while if $m < 0$, the line passes through the II and IV quadrant.
4. A mathematical statement which contains either $<, >, \leq, \geq, or \neq$ is called an inequality.
5. A linear inequality in one variable "x" is an inequality that can be written in the form of ax + b < 0, ax + b ≤ 0, ax + b > 0 or, ax + b ≥ 0 where a and b are rational numbers and $a \neq 0a \neq 0.$
6. Two linear inequalities are said to be equivalent if and only if they have the same solution set.
7. [Rules of transformation]

   For any rational numbers $a, b,$ and c

   i. If $a < b$, then $a + c < b + c$

ii. If $a < b$, then $a - c < b - c$

iii. If $a < b$ and $c > 0$, then $ac < bc$

iv. If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$

v. If $a < b$ and $c < 0$, then $ac > bc$

vi. If $a < b$ and $c < 0$, then $\frac{a}{c} > \frac{b}{c}$

REVIEW EXERCISE

1. Write true for correct statement and false for the incorrect one.

   a. The graph of the line $y = -2x$ passes through the II and III quadrants.

   b. The graph of the equation $x = a,\ a \in \mathbb{Q}, a \neq 0$, if $a > 0$ is a vertical line that lies to the right of the y-axis.

   c. A horizontal line has the equation $x - b = 0$.

   d. If $a < b$ and $c = -3$, then $ac > bc$

   e. The inequality $8 \geq 5x,\ x \in$ W has a finite solution set.

2. Plot the following points in a Cartesian coordinate plane: (0, 6), (2, −3), (−4, 5) and (−4, −5). Which point lies in neither of the quadrant?

3. Refer Figure 3.12 and answer the following questions.

   a. Name the coordinates of the point D, E and F.

   b. Which point has the coordinates (−2, 1)?

   c. Which coordinate of B is 3?

   d. In which quadrant does the point C, E lie?

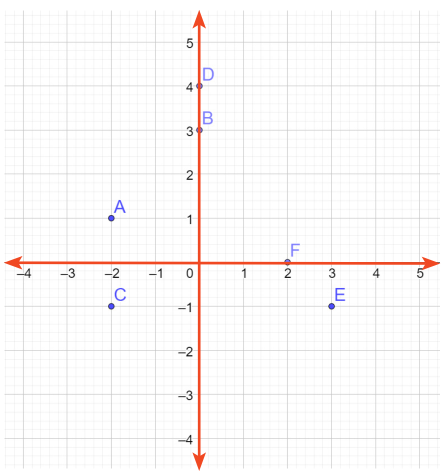


Figure3.12:

4. Fill in the blank with an appropriate inequality sign.

   a. If $x \geq 3$, then $-4x$ ________ -12

   b. If $x + 4 < 2x$, then $\frac{1}{2}x$ ________ 2

   c. If $x < -2$, then $1 - x$ ________ 3

5. Determine whether the given points are on the graph of the equation

   $x - 2y - 1 = 0$

   a. (0, 0)

   b. (1, 0)

   c. $(-1, -1)$

   d. (2, 1)

6. Sketch the graph of the following equations.

   a. $y = -7x$

   b. $y - 11x = 0$

   c. $3y = -7x + 6$

   d. $y - 7 = 0$

7. Find a and b, if the points P(3, 1) and Q(0,2) lie on the graph of

   $ax - by = 6$

8. Consider the equations $y = x + 1$ and $y = 1 - x$.

   a. Determine the values of y for each equation when the values of x are -1, 0 and 1.

   b. Plot the ordered pairs on the Cartesian coordinate plane.

   c. Which point is a common point, called intersection point?

9. Solve the following inequalities and graph the solution set.

   a) $3x + 11 \leq 6x + 8$

   b) $6-2x > x + 9$

   c) $4-3x \leq -\frac{1}{2}(2 + 8x)$

10. Solve the following inequalities

    a. $\frac{x}{7} \geq \frac{3}{14}x - \frac{1}{7}$

    b. $2(\frac{1}{2} - x) < 3(1 + \frac{1}{2}x) + 5$

    c. $\frac{1}{2}x + \frac{1}{3}x - x - 1 \geq 0$

    d. $\frac{3}{4}y + \frac{1}{2}(y - 3) < \frac{y+1}{4}$

    e. $7(y + 1) - y > 2(3y + 4)$

    f. $-4(y - 1) + 3y \geq 1 - y$

11. Solve the following inequalities in the given domain.

    a. $\frac{2}{3}x < 4(4 - x),\ x \in \mathbb{Z}^+$

b. $\frac{1}{6}-\frac{1}{4}x \geq 2+\frac{2}{3}x,\ x \in \mathrm{W}$

c. $2(3y-7)-14 \geq 3(2y-11),\ x \in \mathbb{Q}$

12. Translate the following sentences in to mathematical expressions.

   a. A year ago a father's age exceeded three times his son's age.

   b. Three –fourth of a number is greater than 12.

   c. The average mark of Saron is not smaller than 89.

13. A board with 2.5 m in length must be cut so that one piece is 30 cm more than the other piece. Find the length of each piece.

Figure3.13:

14. Rodas is 25 years old and her brother Mathanya is 10 years old. After how many years will Rodas be exactly twice as old as Mathanya.

15. Yoseph wants to surprise his wife with a birthday party at her favourite restaurant. It will cost Birr 56.50 per person for dinner, including tip and tax. His budget for the party is Birr 735. What is the maximum number of people Yoseph can have at the party?

16. Getaneh and Salhedin play in the same soccer team. Last Saturday Salhedin scored 3 more goals than Getaneh, but together they scored less than 7 goals. What are the possible number of goals Salhedin scored?

Unit 4

4. SIMILARITY OF FIGURES

Learning Outcomes:

At the end of this unit, learners will able to:

- Know the concept of similar figures and related terminologies.
- Understand the condition for triangles being similar.
- Apply tests to check whether two given triangles are similar or not.
- Apply real-life situations in solving geometric problems.

Main content

4.1 Similar plane figures

4.2 Perimeter and Area of Similar Triangles

- Summary
- Review Exercise

4.1. Similar Plane Figures

Competencies:

At the end of this section, students should:

- Identify figures that are similar to each other.
- Apply the definition of similarity of two triangles to solve related problems.
- Determine the similarity of two triangles.

Introduction:

This unit focused on similarity of plane figures. In common language the word similar can have many meanings but in mathematics, the word similar and similarity have very specific meanings. In mathematics, we say that two objects are similar if they have the same shape, but are not necessarily the same size. For instance, an overhead projector forms an image on the screen which has the same shape as the image on the transparency but with the size altered. Two figures that have the same shape but not necessarily the same size are called similar figures.

Activity: 4.1.

1. Which of the following figures are similar?

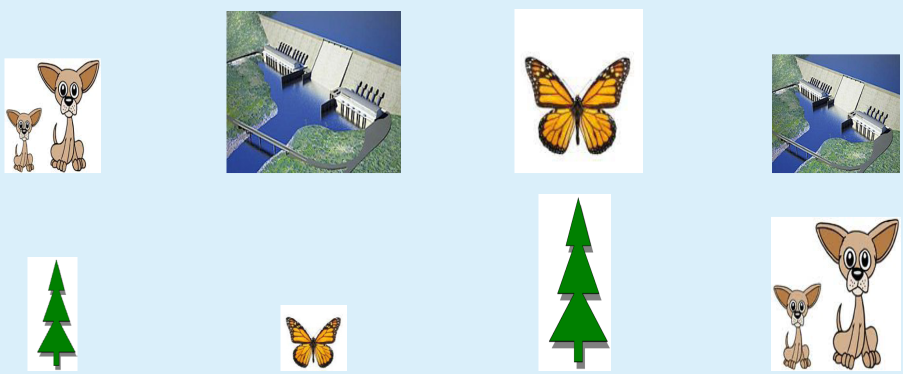


Figure4.1:

2. Are any two equilateral triangles always congruent?
3. Let ABCD be a square and $\overline{AC}$ be its diagonal. Are $\Delta ABC \cong \Delta ADC$?
4. In the triangles, ΔABC and ΔPQR, if $\angle A \cong \angle P$, $\angle B \cong \angle Q$ Then what must be true about $\angle C$ and $\angle R$
5. Decide whether the polygons are similar.
   a. Rectangle and parallelogram
   b. Square and trapezium

4.1.1. Definition and Illustration of Similar Figures

In this section, we will discuss how to compare the size and shape of two given figures. Recall that, one way to determine whether two geometric figures are congruent is to see if one figure can be moved on to the other figure in such a way that it "fits exactly"

If two figures have the same shape, but not necessarily the same size (That is, one figure is an exact scale model of the other figure), then we say that the two geometric figures are similar. The symbol $\sim$ (read as "is similar to") is used for the term similar.

Example 4.1:

A tree and its shadow are similar.

Example 4.2:

The two photograph of the same size of the same person, one at the age of 5 years and the other at the age of 50 years are not similar.

Because similar figures differ only in size, there is a test we can perform to make sure that our shapes are really similar.

Similar Polygons

Definition 4.1

Two polygons are said to be similar if there is a one – to – one correspondence between their vertices such that:

i. all pairs of corresponding angles are congruent

ii. the ratio of the lengths of all pairs of corresponding sides are equal.

In the diagram, ABCD is similar to EFGH.

That is, ABCD $\sim$ EFGH, then

i. $\angle A \cong \angle E$, $\angle B \cong \angle F$, $\angle C \cong \angle G$, $\angle D \cong \angle H$

ii. $\frac{AB}{EF} = \frac{BC}{FC} = \frac{DC}{HG} = \frac{AD}{EH}$

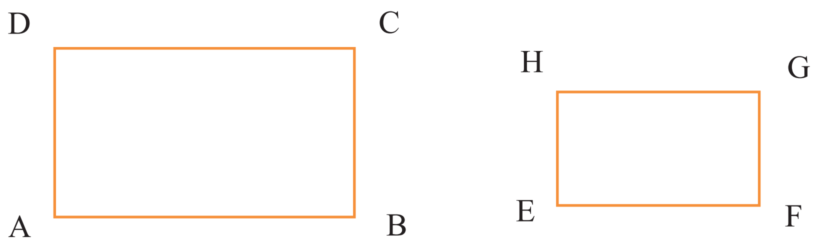


Figure4.2:

Example 4.3:

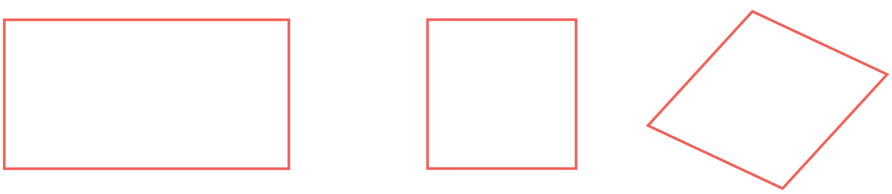
The following pairs of figures are always similar

a. Any two squares.

b. Any two equilateral triangles.

Example 4.4:

Consider the following polygons. Which of these are similar?



Solution:

The square and the rectangle are not similar. Because their corresponding angles are congruent but their corresponding sides are not proportional. The square and the rhombus are not similar. Because their corresponding sides are proportional but their corresponding angles are not congruent. The rectangle and the rhombus are not similar.Because their corresponding angles are not congruent and corresponding sides are not proportional.

Definition:4.2

The ratio of two corresponding sides of similar polygons is called the scale factor or constant of proportionality (k).

Example4.5:

The sides of a quadrilateral are 2cm, 5cm, 6cm and 8cm. Find the sides of a similar quadrilateral whose shortest side is 3cm.

Solution:

Let the corresponding sides of a quadrilateral are 3, x, y, and z. Since the corresponding sides of similar polygons are proportional,

$\frac{2}{3} = \frac{5}{x}$, $\quad \frac{2}{3} = \frac{6}{y}$, $\quad \frac{2}{3} = \frac{8}{z}$

$x = \frac{15}{2}$, $\quad y = 9$, $\quad z = 12$

Therefore, the sides of the second quadrilateral are 3cm, 7.5cm, 9cm, and 12cm.

Example 4.6:

Decide whether the polygons are similar. If so find the scale factor of Figure A to Figure B.

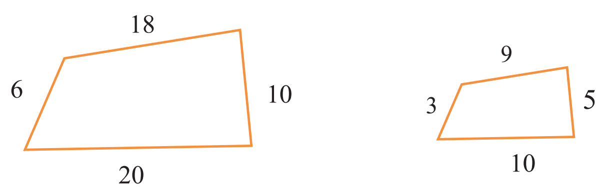


Figure4.3: *a* *b*

Note:	For the similarity of polygons with more than three sides, the two conditions given in the definition are not independent of each other. That is, either of the two conditions without the other is not sufficient to make the polygons similar. Thus, both conditions must be true for polygons to be similar.

Exercise 4.1.

1. Write true if the statement is true or false otherwise.
   a. Any geometric figure is similar to itself.
   b. Congruent polygons are not necessarily similar.
   c. Any two regular polygons that have the same number of sides are similar.
   d. Any two parallelograms are similar.
   e. Two isosceles triangles are similar.
2. What must be the constant of proportionality of two similar polygons in order for the polygons to be congruent?
3. The sides of a quadrilateral measure 12cm, 9cm, 16cm and 20cm. The longest side of a similar quadrilateral measures 8cm. Find the measure of the remaining sides of this quadrilateral.

4.1.2. Similar Triangles

You have defined similar polygons in section 4.1.1. Also you know that, any polygon could be dividing into triangles by drawing the diagonals of the polygon. Thus the definition you gave for similar polygons could be used to define similar triangles.

Definition:4.3

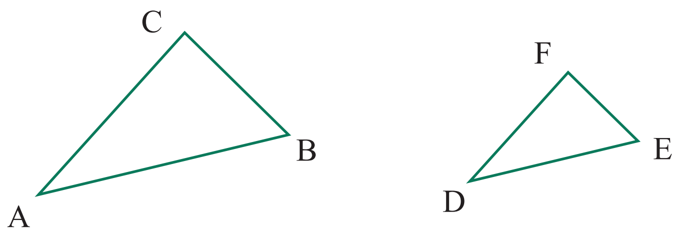


Figure4.4:

Δ ABC is similar to Δ DEF (Δ ABC ~ΔDEF) if their Corresponding:

iii. Sides are proportional.

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = k\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = k$$

iv. Angles are congruent.

$\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$

Example: 4.7:

Show that Δ LMN ~Δ PQR.

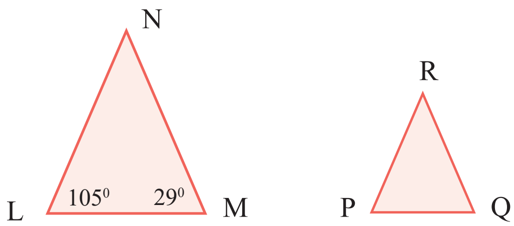


Figure4.5:

Solution:

To show the corresponding angles are congruent, first find the unknown angle.

In Δ LMN, $m\angle L + m\angle M + m\angle N = 180^0$

$105^0 + 29^0 + m\angle N = 180^0$

$m\angle N = 180^0 - 134^0 = 46^0$

In Δ PQR, $m\angle P + m\angle Q + m\angle R = 180^0$

$m\angle P + 29^0 + 46^0 = 180^0$

$m\angle P = 180^0 - 75^0 = 105^0$

Hence, the corresponding angles are congruent.

To show whether corresponding sides are proportional or not, check that the ratio of the corresponding sides is the same.

$$\frac{LM}{PQ} = \frac{10}{4} = 2.5, \quad \frac{MN}{QR} = \frac{20}{8} = 2.5, \quad \frac{LN}{PR} = \frac{15}{6} = 2.5$$

Hence, the corresponding sides are proportional with proportionalityconstant 2.5.

Therefore, $\Delta LMN \sim \Delta PQR$.

Suppose $\Delta ABC \sim \Delta DEF$.

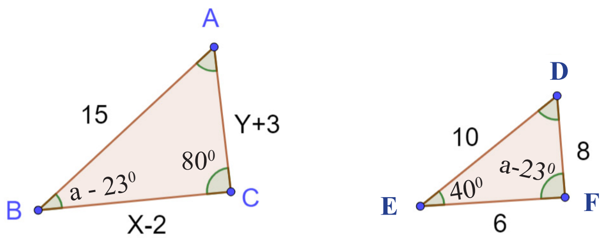


Figure4.6:

Then

a. Find the values of x and y.

b. Find the values of a and b.

Solution:

a. Since $\Delta ABC \sim \Delta DEF$, then $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

$\frac{x-2}{6} = \frac{15}{10}$ $\qquad$ $\frac{15}{10} = \frac{y+3}{8}$

$x - 2 = 6 \times \frac{3}{2}$ $\qquad$ $y + 3 = 8 \times \frac{3}{2}$

$x = 11$ $\qquad$ $y = 12$

b. Since $\Delta ABC \sim \Delta DEF$, then

$m\angle A = m\angle D$ $\qquad$ $m\angle B = m\angle E$

$(a - 23)^0 = 40^0$ $\qquad$ $80^0 = (2b + 10)^0$

$a = 63^0 b = 35^0$

Exercise 4.2.

1. Given that $\Delta ABC \sim \Delta DEF$. Then find x and y.
   a. $\frac{x}{DE} = \frac{AC}{DF}$
   b. A side corresponds to $\overline{CA}$ is x
   c. $\angle B \cong y$
2. Let ABCD be a square.Then show that $\Delta ABC \sim \Delta ADC$.
3. Challenge problem [You may use compass, straightedge, and protractor]
   i. Draw any triangle, ΔABC
   ii. Draw any line segment $\overline{DE}$ with DE = 2AB.
   iii. Construct $\angle GDE \cong \angle A$ and $\angle HED \cong \angle B$.

Let F be the intersection of $\overrightarrow{DG}$ and $\overrightarrow{EH}$.

   a. Is DF = 2AC ? Is EF = 2 BC ?
   b. Is $\Delta DEF \sim \Delta ABC$?

From the above challenge problem, you observe the following. For any given triangle, there exists a similar triangle with any given constant of proportionality.

4.1.3. Tests for similarity of triangles [AA, SSS, and SAS]

We have proved triangles similar by proving that the corresponding angles are congruent and that the ratios of the lengths of corresponding sides are equal, which is long and tiresome. Hence we want to have the minimum requirements which will guarantee us that the triangles are similar. It is possible to prove that when some of these conditions exist, all of these conditions necessary for triangles to be similar exist. These short cut techniques are given as similarity theorems. These similarity theorems help us quickly find out whether two triangles are similar or not.

Activity: 4.2.

Melaku said that if an acute angle of one right triangle is congruent to an acute angle of another right triangle, the triangles are similar. Do you agree with Melaku? Explain why or why not.

Theorem 4.1.

[AA- Similarity Theorem]

If two angles of one triangle are congruent to the corresponding two angles of another triangle, then the two triangles are similar.

Thus, $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\Delta ABC \sim \Delta DEF$

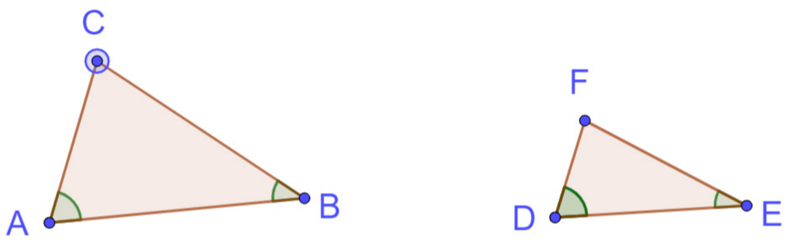


Figure4.7:

Example: 4.9:

Suppose $\overline{DE} \parallel \overline{BC}$ as shown in the figure. Then show that $\angle A \cong \angle A$?

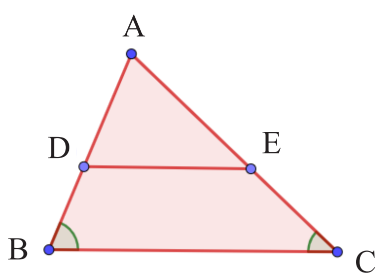


Figure4.8:

Proof:

	Statements	Reasons
1.	$\angle A \cong \angle A$	Common angle
2.	$\angle B \cong \angle D$	Corresponding angles
3.	$\Delta ADE \sim \Delta ABC$	AA similarity theorem

From the above example we conclude that a line through two sides of a triangle parallel to the third side cut off a triangle similar to the given triangle.

Similar triangles are very useful for indirectly determining the size of items which are difficult to measure by hand.

Example 4.10:

Mustefa is a 160 cm tall. He is standing 350 cm away from a lamp post and his shadow is 80cm long. How high is the lamp post?

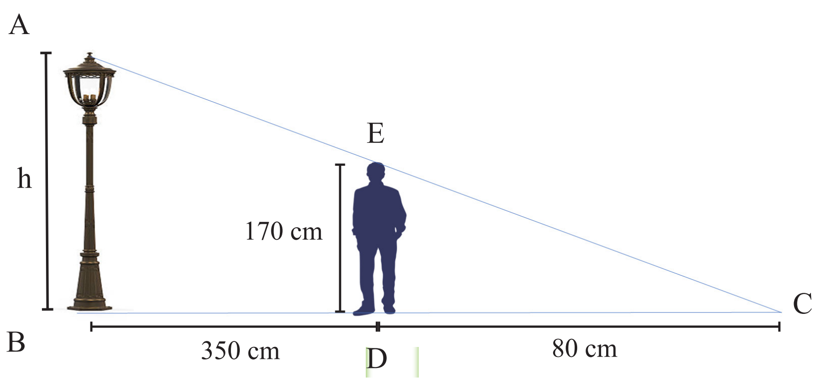


Figure4.9:

Solution:

Consider ΔABC and ΔEDC. We can see that $\angle B \cong \angle D = 90^0$ and $\angle C \cong \angle C$ (common angle). Hence, by AA similarity, $\Delta ABC \sim \Delta EDC$.

$\frac{AB}{DE} = \frac{BC}{DC} = \frac{AC}{EC}$ $\qquad$ $\frac{h}{160} = \frac{350}{80}$ $\qquad$ $h = 700$ cm

Therefore, the height of the lamp post is 700 cm long.

Example 4.11:

Consider the figure below. Determine the values of x and y.

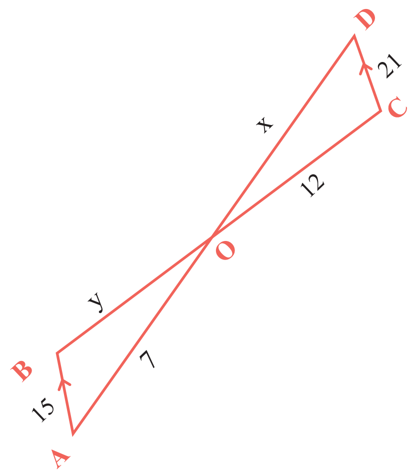


Figure4.10:

Solution:

$\angle B \cong \angle C$ [Alternate Interior Angles]

$\angle O \cong \angle O$ [Vertically Opposite Angles]

Then, by AA similarity theorem,

$\Delta ABO \sim \Delta DCO$.

Thus, $\frac{AB}{DC} = \frac{BO}{CO} = \frac{AO}{DO}$

$\frac{15}{21} = \frac{y}{12}$ and $\frac{15}{21} = \frac{7}{x}$

$y = \frac{60}{7}$ and $x = \frac{49}{5}$

Theorem 4.2.

[SAS similarity theorem]

If an angle of one triangle is congruent to an angle of another triangle and the sides including those angles are proportional, then the triangles are similar.

Thus, if $\angle C \cong \angle F$, $\frac{AC}{DE} = \frac{BC}{EF}$, then $\Delta ABC \sim \Delta DEF$.

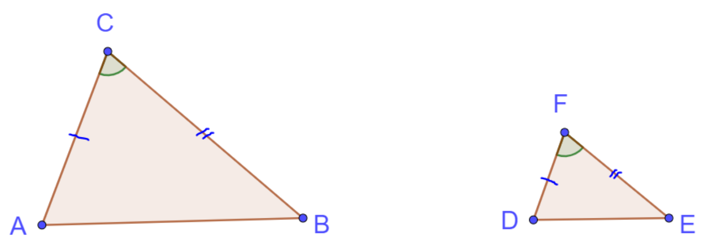


Figure4.11:

Example 4.12:

Is $\Delta ABC \sim \Delta DEC$? Explain.

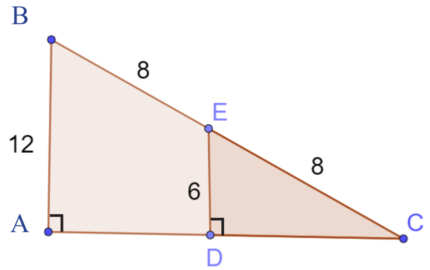


Figure4.12:

Solution:

$\frac{AB}{DE} = \frac{12}{6}$ *and* $\frac{BC}{EC} = \frac{16}{8} = 2$ .... sides are proportional

$m(\angle B) = m(\angle E) = 90^0$ ….. included angles are congruent

Therefore, by SAS similarity theorem Δ ABC ~ Δ DEC

Example 4.13:

In the figure below,

i. Show that Δ ABC ~ Δ EFD.

ii. Find the value of x.

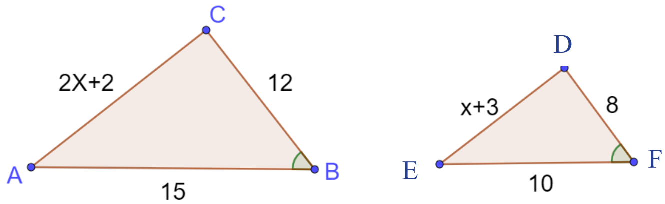


Figure4.13:

Solution:

i. $\frac{AB}{EF} = \frac{15}{10} = \frac{3}{2}$ and $\frac{BC}{FD} = \frac{12}{8} = \frac{3}{2}$.

$\angle B \cong \angle F$ …… included angle

Therefore, Δ ABC ~ Δ EFD by SAS similarity theorem.

ii. $\frac{AB}{EF} = \frac{BC}{EF}$ $\qquad$ $\frac{2x+2}{x+3} = \frac{3}{2}$

$x = 5$

Theorem 4.3.

[SSS Similarity Theorem]

If the three sides of one triangle are in proportion to the three sides of another triangle, then the two triangles are similar.

Thus, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$, then Δ ABC ~ Δ DEF

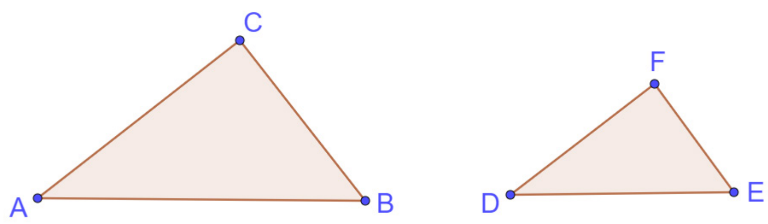


Figure4.14:

Example 4.14:

Based on the given figure show that Δ ABC ~ Δ EFD.

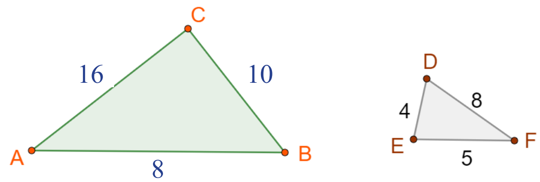


Figure4.15:

Solution:

$\frac{AB}{DE} = \frac{8}{4} = 2,\qquad \frac{BC}{EF} = \frac{10}{5} = 2,\qquad \frac{AC}{DF} = \frac{16}{8} = 2$

Hence, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ [the corresponding sides are proportional]

Therefore, Δ ABC ~ Δ EFD …… SSS Similarity Theorem.

Example 4.15:

The length of the sides of Δ EFD are EF = 15cm, FD = 8cm and ED = 12cm. If Δ EFD ~Δ LMN and the length of the smallest side of Δ LMN is 6cm, find the measures of the other two sides of Δ LMN.

Solution:

Since the smallest side of Δ EFD is FD = 8cm and $\overline{FD}$ corresponds to $\overline{MN}$, MN = 6cm.

$\frac{FD}{MN} = \frac{EF}{LM}$,

$\frac{8}{6} = \frac{15}{LM}$

LM = $11\frac{1}{4}$cm

$\frac{FD}{MN} = \frac{ED}{LN}$,

$\frac{8}{6} = \frac{12}{LN}$

LN = 9cm

Therefore, the sides of Δ LMN are 6cm, $11\frac{1}{4}$cm and 9cm.

Example 4.16:

A man who is 1.80m tall wishes to find the height of a certain four-story building. He measures its shadow and finds it to be 28m long; while his own shadow is 4.2m long. How tall is the building?

Solution:

Let h = the height of the building.

Draw a line from top of a building to the end of its shadow. Similarly draw a parallel line through the head of man and its shadow.

Ratio of height to base in large triangle = $\frac{h}{28}$

Ratio of height to base in small triangle = $\frac{1.8}{4.2}$

Since the large and small triangles are similar (Which similarity theorem?), we get the equation $\frac{h}{28} = \frac{1.8}{4.2}$

Now we solve for h,

$$h = \frac{1.8 \times 28}{4.2}$$

$$= 12$$

So the building is 12 m tall.

Exercise 4.3.

1. In the figure below, if $\overline{AB} \parallel \overline{ED}$, AC = 5,AB = 6 and ED = 3 then
   a. Show that Δ ACB ~ Δ ECD.
   b. Find the values of x and y.

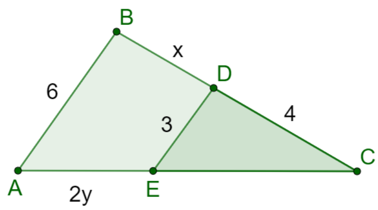


Figure4.16:

2. Let Δ LMN ~ Δ PQR and the sides of Δ LMN are LM = 12, MN = 15, and LN = 18. If the constant of proportionality is $\frac{2}{3}$, then find the length of the corresponding sides of Δ PQR.
3. When you shine a flashlight on a book that is 12cm tall and 8cm wide, it makes a shadow on the wall that is 18cm tall and 12cm wide. Then what is the scale factor of the book to its shadow?
4. Given the figure below.

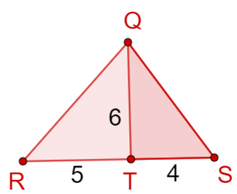


Figure4.17:

   a. Show that Δ RSQ ~ Δ RQT
   b. What is the scale factor of Δ RSQ to Δ RQT?
   c. Find the length of $\overline{SQ}$.
   d. Let RS = 9cm. Is Δ RSQ similar to Δ QST? Explain
5. Δ PQR and Δ LMN are similar such that ∠P ≅ ∠L, ∠Q≅ ∠M, LM = 8, MN = 10, PQ = 12, and PR = 18. Find the lengths of LN and QR.
6. The sides of a triangle measures 4cm, 9cm and 11cm. If the shortest side a similar triangle measures 12cm, find the measure of the remaining sides of this triangle.

4.2. Perimeter and Area of Similar Triangles

Competency: **At the end of this section, students should:**

- Explain the relation between the perimeters of two similar triangles.
- Explain the relation between the areas of two similar triangles.

Activity: 4.3.

1. What does Perimeter of a triangle mean? Area of a triangle mean?
2. How do you calculate the area of a right- angled triangle?
3. Suppose $\triangle RPQ \sim \triangle DEF$ as shown below.
   a. Find the perimeter P_1 of $\triangle RPQ$ and perimeter P_2 of $\triangle DEF$.
   b. Find $\frac{P_1}{P_2}$ [perimeter ratio]
   c. Compare $\frac{P_1}{P_2}$ and $\frac{S_1}{S_2}$ [side ratio or scale factor]
   d. Find the area A_1 of $\triangle RPQ$ and A_2 of $\triangle DEF$
   e. Find $\frac{A_1}{A_2}$ [area ratio]
   f. Compare $\frac{A_1}{A_2}$ and $(\frac{S_1}{S_2})^2$
   g. What do you observe?

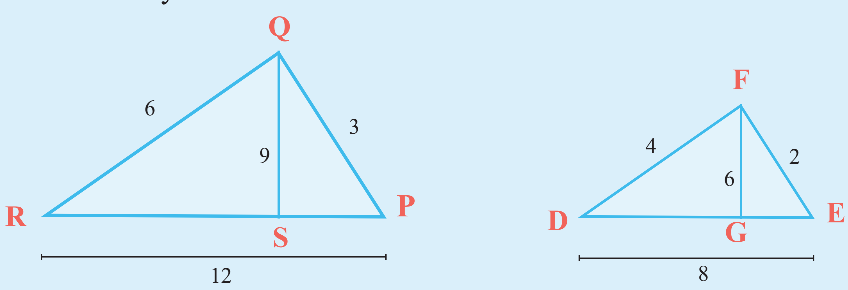


Figure4.18:

From grade 7 mathematics, you recall that the triangle and the set of points inside of it is called a triangular region, and the area of this triangular region refer the area of a triangle.

Example 4.16:

Let Δ ABC~ Δ PQR and $\overline{CD}$ and $\overline{RE}$ are the respective altitudes from vertex C and vertex R of the triangles.

Then determine the relationship between:

a. The altitudes of the triangles.
b. The perimeters of the triangles.
c. The areas of the triangles.

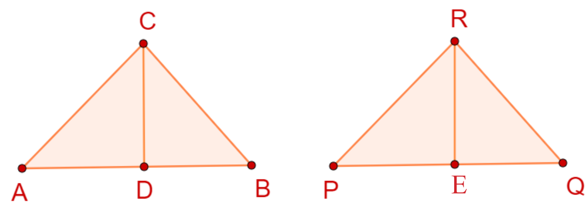


Figure4.19:

Solution:

a. Δ ABC~ Δ PQR

Let k be the constant of proportionality between the corresponding sides. That is, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = k$.

$AB = kPQ, \quad BC = kQR, \quad AC = kPR$

Since $\overline{CD}$ and $\overline{RE}$ are the altitudes, ∠D ≅ ∠E [right angles] and

∠B ≅ ∠Q [Similarity definition].

Hence Δ CDB ~ Δ REQ by AA similarity.

Therefore, $\frac{CD}{RE} = \frac{CB}{RQ} = k$.

$CD = k.RE$

b. p(Δ ABC) = $AB + BC + CA = P_1$

$= kPQ + kQR + kRP$

$= k(PQ + QR + RP)$ and

p(Δ PQR) = $PQ + QR + RP = P_2$

Then, p(Δ ABC) = k p(Δ PQR)

$P_1 = k\,P_2$

(Δ ABC) = $\frac{1}{2}$.AB. CD $= A_1$

$= \frac{1}{2}.k$PQ. kRE

$= k^2.\frac{1}{2}$.PQ. RE and

a(Δ PQR) $= \frac{1}{2}.$ PQ. RE $= A_2$

Then, a(Δ ABC) = k^2 .a(Δ PQR)

$$A_1 = k^2 A_2$$

The above example will lead us to the following generalizations.

Theorem 4.4.

If the ratios of the corresponding sides of two similar triangles is k, then the ratio of their perimeters is given by: $\frac{P_1}{P_2} = \frac{S_1}{S_2} = k$

Theorem 4.5.

If the ratios of the corresponding sides of two similar triangles is k, then the ratio of their areas is given by: $\frac{A_1}{A_2} = (\frac{S_1}{S_2})^2 = k^2$

Example4.17:

The corresponding sides of two similar triangles are 8cm and 6cm. Find the ratio of

a. The perimeters of the triangles

b. The areas of the triangles.

Solution:

$\frac{S_1}{S_2} = \frac{8}{6} = \frac{4}{3}.$

a. $\frac{P_1}{P_2} = \frac{S_1}{S_2} = \frac{4}{3}$

b. $\frac{A_1}{A_2} = \left(\frac{S_1}{S_2}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$

Example 4.18:

The areas of two similar triangles are 64 cm^2 and 100cm^2. If one side of the smaller triangle is 4cm, then find the corresponding side of the second triangle.

Solution:

Let $A_1 = 64\ cm^2$, $A_2 = 100\ cm^2$, and $S_1 = 4cm$

Then, $\frac{A_1}{A_2} = \left(\frac{S_1}{S_2}\right)^2$

$\frac{64}{100} = \left(\frac{4}{S_2}\right)^2$

$S_2{}^2 = \frac{100 \times 16}{64}$

$S_2 = 5cm$

Example 4.19:

The ratio of the sides of two similar triangles is $\frac{2}{3}$. The area of the smaller is triangle is 36cm^2. Find the area of the larger triangle.

Solution:

Let S_1 be the sides of the smaller triangle whose corresponding area be A_1.

$\frac{S_1}{S_2} = \frac{2}{3}$, and $A_1 = 36cm^2$. Then,

$\frac{A_1}{A_2} = \left(\frac{S_1}{S_2}\right)^2$

$\frac{36}{A_2} = \left(\frac{2}{3}\right)^2$

$A_2 = \frac{36 \times 9}{4} = 81cm^2$

Example 4.20:

The lengths of sides of a triangle are 16cm, 23cm, and 31cm. If the perimeter of a similar triangle is 280cm, find

a. The length of the longest side of the second triangle

b. The ratio of the area of the largest triangle to the smaller.

Solution:

a. Let P_1 be the perimeter of the first triangle. Then

$P_1 = 16 + 23 + 31 = 70cm$, $P_2 = 280cm$

$\frac{P_1}{P_2} = \frac{S_1}{S_2}$ $\frac{70}{280} = \frac{31}{S_2}$

$S_2 = \frac{280 \times 31}{70} = 124cm$

b. $\frac{A_2}{A_1} = \left(\frac{P_2}{P_1}\right)^2 = \left(\frac{280}{70}\right)^2 = 16$

Therefore, $A_2 : A_1 = 16:1$

Exercise 4.4.

1. In Figure 4.20 given below, find the ratio of
   a. Perimeters of Δ ABE to Δ ACD
   b. Areas of Δ ABE to Δ ACD

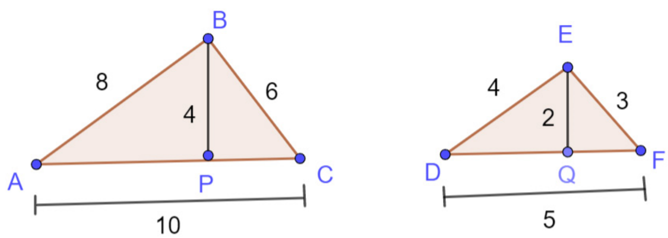


Figure4.20:

2. The scale factor of Δ ABC to Δ DEF is 4 : 3. The area of Δ ABC is x and the area of Δ DEF is 7. The perimeter of Δ ABC is 8 + y and the perimeter of Δ DEF is $3y - 12$.
   a. Find the perimeter of Δ DEF.
   b. Find the area of Δ ABC.
3. Δ ABC ~ Δ DEF. The length of altitude $\overline{BP}$ exceeds the length of altitude $\overline{EQ}$ by 7. If AC = $\frac{5}{4}$ DF, then find the length of each altitude.

UNIT SUMMARY

1. Similar geometric figures are figures which have the same shape, but not necessarily the same size.
2. Two polygons are similar if:
   i. Their corresponding sides are proportional.
   ii. Their corresponding angles are congruent.
3. Congruent figures are always similar.
4. The ratio of two corresponding sides of similar polygons is called the scale factor or constant of proportionality (kk).
5. Δ ABC is similar to Δ DEF, Δ ABC$\sim$ Δ DEF if
   I. Their corresponding sides are proportional.
   II. Their corresponding angles are congruent.
6. **AA similarity theorem:** If two angles of one triangle are congruent to the corresponding two angles of another triangle, then the two triangles are similar.
7. **SAS similarity theorem:** If an angle of one triangle is congruent to an angle of another triangle and the sides including those angles are proportional, then the triangles are similar.
8. **SSS similarity theorem:** If the three sides of one triangle are in proportion to the three sides of another triangle, then the two triangles are similar.
9. If Δ ABC $\sim$ Δ DEF with constant of proportionality k, then
   i. p(Δ ABC) = k. p(Δ DEF)
   ii. a(Δ ABC) = k^2.(Δ Δ DEF)

REVIEW EXERCISE

1. Write true if the statement is correct and false if it is not.
   a. Two congruent polygons necessarily similar.
   b. Two similar polygons are necessarily congruent.

c. Congruence is a similarity when the constant of proportionality is 1.

d. Rectangle ABCD is similar to rectangle EFGH if $\frac{AB}{EF} = \frac{BC}{FG}$

e. Doubling the side length of a triangle doubles the area.

f. Two triangles with the same area are similar.

2. Let Δ ABC ~ Δ DEF and the ratio of the respective altitudes is 1:3. If the measures of the sides of Δ Δ ABC are 3cm, 5cm and 7cm , then find the measures of the sides of the larger triangle, Δ Δ DEF.

3. In Δ ABC, the midpoint of $\overline{AC}$ is M and the midpoint of $\overline{BC}$ is N.

a. Show that Δ ABC ~ Δ MNC.

b. What is the ratio of the sides of Δ MNC to Δ ABC.

4. In Δ ABC shown below, $\overline{DE} \parallel \overline{AC}$. Find the values of x and y

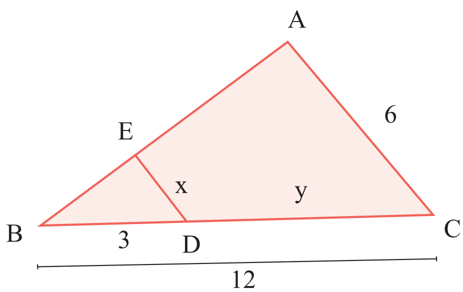
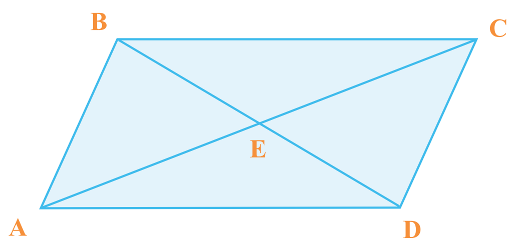


Figure4.21:

5. The 1.80m tall lady makes a 9m long shadow, and the palm tree makes a 26m long shadow. Find the height of the tree. [Draw a diagram by yourself]

6. ABCD is a parallelogram with parallel sides $\overline{BC}$ and $\overline{AD}$ and its diagonals $\overline{BD}$ and $\overline{AC}$ intersect at E. Show that Δ BEC ~ Δ AED

Figure4.22:



7. In the figure below, Δ ABC ~ Δ DEF by AA similarity theorem. If $\angle ABC \cong \angle DEF$, $\angle ACB \cong \angle DFE$ $AB = 8cm$, $AC = 10cm$ and $DE = 10cm$ Find the length of FE.

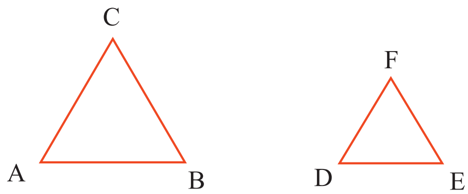


Figure4.23:

8. What pair of congruent angles and what proportion are needed to prove $\Delta ADE \sim \Delta ABC$?

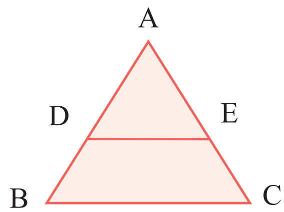


Figure4.24:

9. Indicate the proportion needed to prove $\Delta ADE \sim \Delta ABC$.

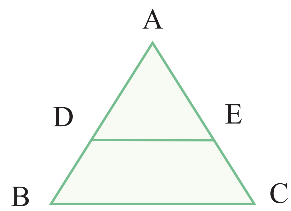


Figure4.25:

10. What angles can be used to prove $\Delta AED \sim \Delta FGB$ (ABCD is a parallelogram)

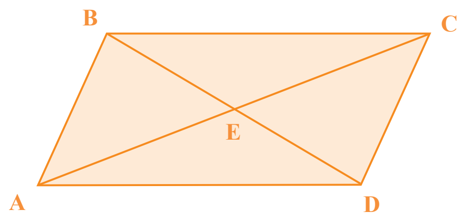


Figure4.26:

11. The areas of two similar triangles are 64 cm^2 and 144cm^2.

    a. find the ratio of their altitudes.

    b. If the perimeter of the smaller triangle is 49cm, find the perimeter of the larger triangle.

Unit 5

5. THEOREMS ON TRIANGLES

Learning Outcomes:

At the end of this unit, learners will able to:

- Understand basic concepts about right angled triangles
- Apply some important theorems on right angled triangles.
- Apply real-life situations in solving geometric problems

Main content

5.1 The three angles of a triangle add up to 180°

5.2 The exterior angle of a triangle equals the sum of the two remote angles

5.3 Theorems on the right angled triangle

- Summary
- Review Exercise

Introduction

In geometry, one of the most used shapes is a triangle. A triangle has three sides and three angles. These sides and angles are the elements of the triangle. All the polygons have two types of angles which are interior angles and exterior angles. As the triangle is the smallest polygon, it has three interior angles and six exterior angles. There are various kinds of triangles with different angles and edges, but all of them follow the triangle sum properties. The two most important properties are the angle sum property of a triangle and the exterior angle property of a triangle. In this unit you will learn some basic theorems about triangles.

5.1. The three angles of a triangle add up to 180^0

Competencies: **At the end of this sub-topic, students should:**

- Describe the angle sum theorem of a triangle
- Apply the angle sum theorem of a triangle in solving related problems

Group work 5.1

1. Define each of the following terms in your own words.
   a. Vertically opposite angles
   b. Alternate interior angles
   c. Alternate Exterior angles
   d. Corresponding angles
2. Calculate the measures of marked angles

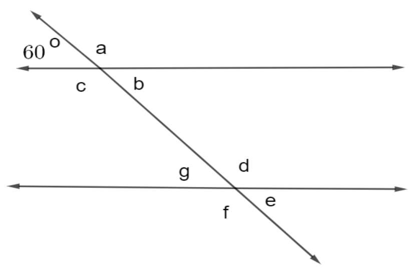


Figure5.1:

3.

a. Take $\overline{AB}$. Fold $\overline{AB}$ to get the midpoint at D. Similarly take $\overline{AC}$ and fold to get the midpoint at E.

b. From D and E draw perpendicular line to $\overline{BC}$ whose respective intersection points are F and G.

c. Fold in wards ΔADE at $\overline{DE}$, ΔDBF at $\overline{DF}$ and ΔCEG at $\overline{EG}$

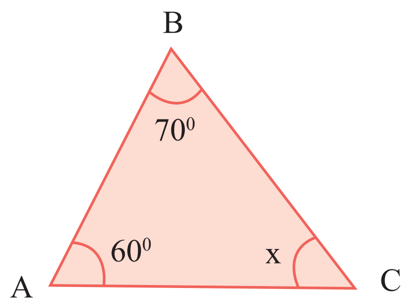
d. Is the three folded triangles meet at a single point? If so say H.

e. Show that $m\angle DHF + m\angle DHE + m\angle EHG = 180^0$

4. From question number 3 observation, find the values of x for the following questions.

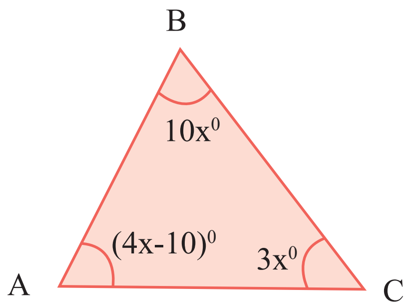
a. What is the value of x in figure 5.2?

Figure5.2:



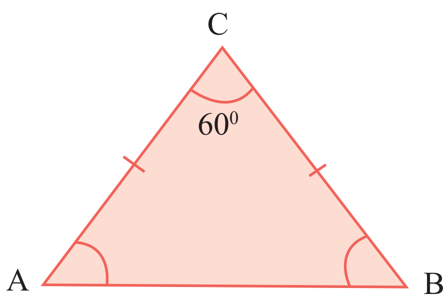
b. What is the measure of ∠B in the figure 5.3?

Figure5.3:



c. Calculate the measure of ∠A in the fig. 5.4

Figure5.4:



Theorem 5.1. **(Angle- Sum Theorem)**

The sum of the degree measures of the interior angles of a triangle is equal to 180°

Proof: Consider the following figure:

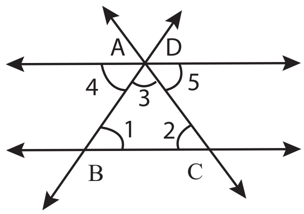


Figure5.5:

Given: ΔABC with $\overline{AD} \parallel \overline{BC}$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180°$

Statements	Reasons
1. ΔABC With $\overline{AD} \parallel \overline{BC}$	Given
2. $\angle 4 \cong \angle 1, \ \angle 5 \cong \angle 2$	Alternate interior angles
3. $m\angle 4 + m(\angle BAD) = 180$	Definition of straight angle.
4. $m\angle 3 + m\angle 5 = m(\angle BAD)$	Addition
5. $m\angle 4 + m\angle 3 + m\angle 5 = 180^0$	Substitution
6. $m\angle 1 + m\angle 2 + m\angle 3 = 180^0$	Substitution

Example5.1:

Find the values of x for each of the following triangles

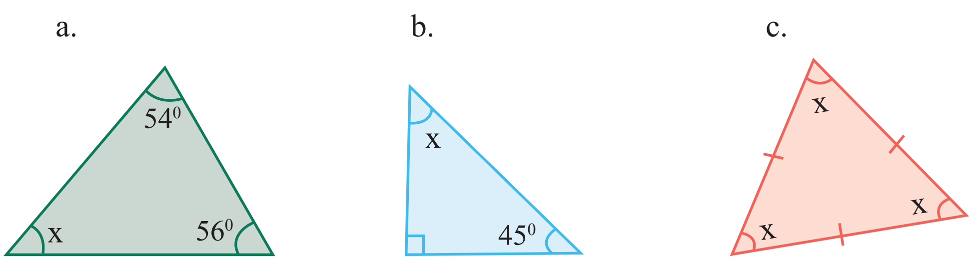


Figure5.6:

Solution:

a. $x + 56° + 54° = 180°$ (Angle Sum Theorem)

$x + 110° = 180°$

$x + 110° - 110° = 180° - 110°$

$x = 70°$

b. Since the triangle is an isosceles right angle triangle

$x + 90° + 45° = 180°$

$x + 135° = 180°$

$x = 45°$

c. Since the triangle is an equilateral triangle

$x + x + x = 180°$

$3x = 180°$

$3x = 180°$

$x = 60°$

Example5.2:

If the measure of the angles of a triangle are $3x$, $4x$,and $5x$, then find the measure of each angle.

Solution:

Consider ΔABC with interior angle measures $3x$, $4x$, and $5x$

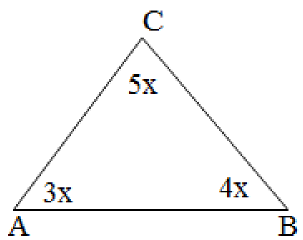


Figure5.7:

Then, $m\angle ABC + m\angle BCA + m\angle CAB = 180^0$ (Angle Sum Theorem)

$4x + 5x + 3x = 180^0$

$12x = 180^0 \quad x = 15^0$

Hence, $m\angle ABC = 4x = 60^0$

$m\angle BCA = 5x = 75^0$

$m\angle CAB = 3x = 45^0$

Therefore, the angles are 45^0, 60^0, and 75^0

Exercise 5.1.

1. Find the values of x for each of the following triangles

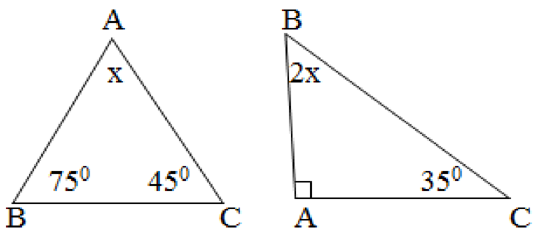


Figure5.8:

2. The measures of the three interior angles of a triangle are in the ratio $2:3:4$.What is the measure of the largest interior angle of a triangle?
3. In a right angled triangle, the measure of one acute angle of a triangle is two times the measure of the other acute angle. Find the measure of each acute angle.
4. Determine the values of x, y and z in the figure below

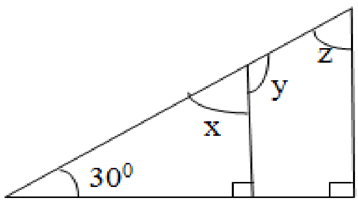


Figure5.9:

5. Find the measures of the missing angles

a.

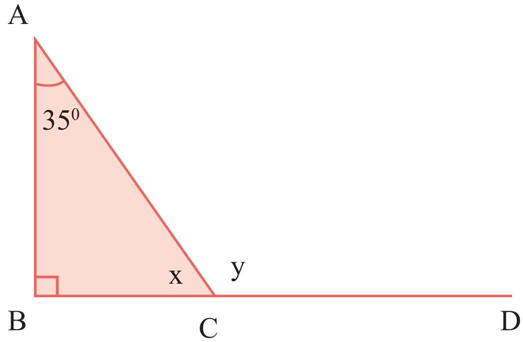


Figure5.10:

b.

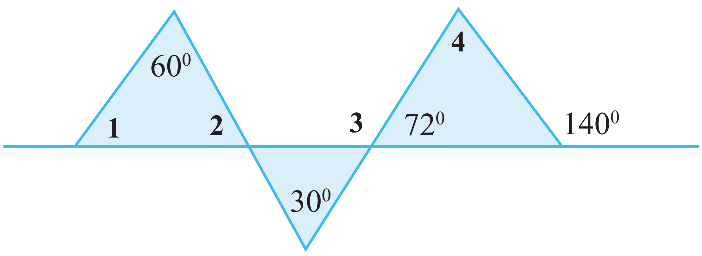


Figure5.11:

5.2. The exterior angle of a triangle equals to the sum of two remote interior angles

Competencies:

At the end of this sub-unit, students should:

- Describe the relation between the exterior angle and the two remote interior angles of a triangle
- Prove the exterior angle of a triangle equals the sum of the two remote interior angles

Activity: 5.1.

1. Define straight angle in your own words.
2. What does an exterior angle of a triangle mean?
3. How do you get an exterior angle of a triangle?
4. Prove that the exterior angle of a triangle equals to the sum of two remote interior angles.

Exterior angle is the angle between one side of the triangle and the extended adjacent side. Each edge of a triangle can form two exterior angles with the two of its extended adjacent sides.Thus, a triangle has six exterior angles.

Theorem 5.2

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

Proof: Let ABC be a triangle with $\overline{AC}$ extended to form an exterior angle. Let x, y and z be the degree measures of the interior angles of Δ ABC and d be degree measure of the formed exterior angle.

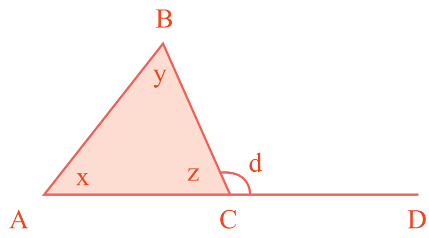


Figure5.12:

We want to show that: $x + y = d$

Statements	Reasons
1. $z + d = 180°$	Straight angle
2. $x + y + z = 180°$	Angle Sum Theorem
3. $x + y + z = z + d$	Substitution
4. 4. $x + y = d$	Subtracting z from both sides

Therefore, the exterior angle of a triangle is equal to the sum of the two remote interior angles.

Example5.3:

Given that for a triangle, the two interior angles 25° and $(x+15)^\circ$ are remote to an exterior angle$(3x-10)^\circ$ .Find the value of x

Solution:

Apply the triangle exterior angle theorem

$(3x-10)^\circ = (25^\circ) + (x+15)^\circ$,

$(3x-10)^\circ = x + 25^\circ + 15^\circ$

$(3x-10)^\circ = x + 40^\circ$

$2x = 50^\circ$

$x = 25^\circ$

Example5.4:

Calculate values of x and y in the following triangle.

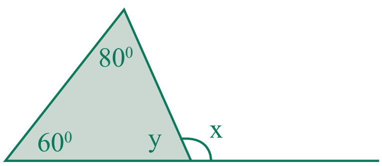


Figure5.13:

Solution:

It is clear from the figure that y is an interior angle and x is an exterior angle

So, by triangle exterior angle theorem

$x = 80^\circ + 60^\circ$

$x = 140^\circ$ and

$y + x = 180^\circ$

$y + 140^\circ = 180^\circ$

$y = 40^\circ$

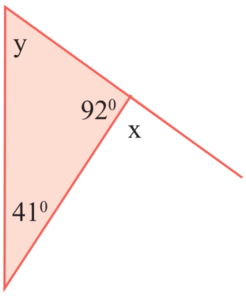
Therefore, the values of x and y are respectively 140° and 40°.

Exercise 5.2.

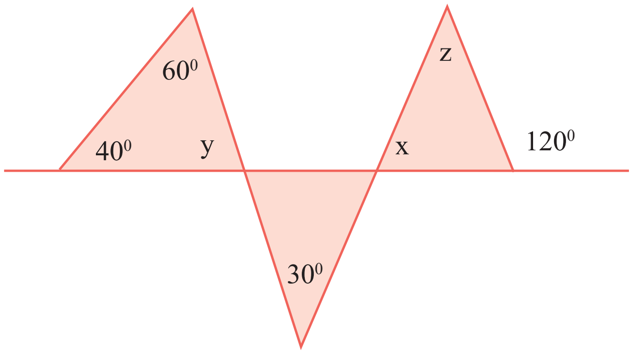
1. The exterior angle of a triangle is 120^0. Find the value of x if the interior remote angles are $(4x + 40)^0$ and 60^0.

2. Calculate the values of x, y and z in the following triangle

a.



b.



c.

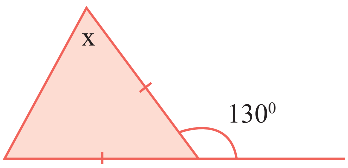


Figure5.14:

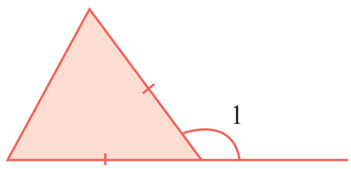
3. Fill in the missing steps of the following exterior angle theorem proof.

Given: $\angle 1$ is an exterior angle of $\triangle ABC$

Prove:

$m\angle 1 = m\angle A + m\angle B$

Figure5.15:



Statements	Reasons
$\angle 1 \angle 1$is an interior angle of $\Delta ABC \Delta ABC$	Given
$m\angle ACB + m\angle 1 = 180^0$	?
?	Triangle Sum Theorem
$m\angle ACB + m\angle 1 = m\angle A + m\angle B + m\angle ACB$	?
$m\angle 1 = m\angle A + m\angle B$	?

5.3. Theorems on the right angled triangle

5.3.1. Euclid's Theorem and its Converse

Euclid's Historical note

Euclid is a famous mathematician, very little is known about his life. Euclid was born around 365 B.C. Euclid's most famous work is his collection of 13 books, dealing with geometry, called the Elements. Euclid is often referred to as the "Father of Geometry".

Figure5.16: (Euclid's)

Competencies:

At the end of this sub-section, students should:

- Describe the right angle triangle, the altitude and the hypotenuse
- Apply Euclid's theorem and its converse for solving related problems

Activity: 5.2.

1. Name the altitude, hypotenuse and the right angle triangle from the given figure5.17 triangle
2. In figure5.17 find three similar triangles

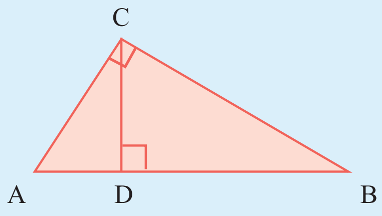


Figure5.17:

In a right triangle, the side opposite the right angle is called the hypotenuse; each of the remaining sides is called a leg. It is sometimes convenient to refer to the length of a side of a triangle by using the lower case letter of the vertex that lies opposite the side. If we draw an altitude to the hypotenuse of the right triangle, we notice that the altitude divides the hypotenuse into two segments.

Theorem 5.3. **(Euclid's Theorem)**

In a right angled triangle with an altitude to the hypotenuse, the square of the length of each leg of a triangle is equal to the product of the hypotenuse and the length of the adjacent segment into which the altitude divides the hypotenuse. Symbolically

1. $(BC)^2 = AB \times BD$ or $a^2 = c \times c_2$
2. $(AC)^2 = AB \times AD$ or $b^2 = c \times c_1$

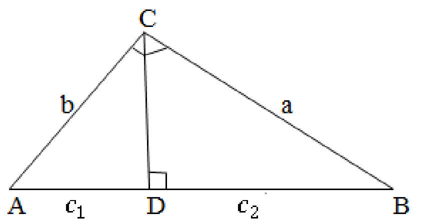


Figure5.18:

Let us consider Figure 5.19 to prove Euclid's theorem.

The altitude $\overline{CD}$ of ΔABC divides the triangle into two right angled triangles: ΔCBD andΔACD. How many right angled triangles do we see in figure 5.19? There are three right angled triangles: ΔACB, ΔCDB and ΔCDA. As a result of the similarity relationships that exist between these pairs of triangles the Euclid's theorem will be proved.

i. ΔCBD ~△ ABC…. (by AA similarity theorem)

$$\frac{CB}{AB} = \frac{BD}{BC}$$

$$\frac{a}{c} = \frac{c_2}{a} \qquad a^2 = cc_2$$

ii. $\Delta ACD \sim \Delta ABC$.... (by AA similarity theorem)

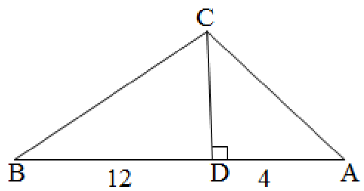
$$\frac{AC}{AB} = \frac{AD}{AC}$$

$$\frac{b}{c} = \frac{c_1}{b}$$ $b^2 = cc_1$ Hence the theorem is proved.

Example 5.5:

In Figure 5.20 to the right, ΔABC is a right angled triangle with $\overline{CD}$ the altitude to the hypotenuse. Determine the lengths of $\overline{AC}$ and $\overline{BC}$

if AD = 4cm and DB = 12cm



Solution:

i. $(AC)^2 = AB \times AD$....... Euclid's Theorem

$(AC)^2 = 16cm \times 4cm$..... Since AB = AD + BD

$(AC)^2 = 64cm^2$

$AC = \sqrt{64cm^2}$

$AC = 8cm$

ii. $(BC)^2 = AB \times BD(BC)^2 = AB \times BD$....... Euclid's Theorem

$(BC)^2 = 16cm \times 12cm$

$(BC)^2 = 192cm^2$

$BC = \sqrt{192cm^2}$

$BC = 8\sqrt{3}cm$

Example5.6:

ΔABC is a right angled triangle with hypotenuse $\overline{AB}$ and altitude $\overline{CD}$ to $\overline{AB}$. If AD = 4cm, DB = 5cm, then find the length of $\overline{AC}$ and $\overline{BC}$.

Solution:

i. $(AC)^2 = AB \times AD$……… Euclid's Theorem

$(AC)^2 = 9cm \times 4cm$ Since AB = AD + BD

$(AC)^2 = 36cm^2$

$AC = \sqrt{36cm^2}$

$AC = 6cm$

ii. $(BC)^2 = AB \times BD$……… Euclid's Theorem

$(BC)^2 = 9cm \times 5cm$ Since AB = AD + BD

$(BC)^2 = 45cm^2$

$BC = \sqrt{45cm^2}$

$BC = 3\sqrt{5}cm$

Theorem 5.4 (Converse of Euclid's Theorem)

In a triangle, if square of each shorter side of a triangle is equal to the product of the length of the longest side of the triangle and the adjacent segment into which the altitude to the longest sides divides this side, then the triangle is right angled.

Symbolically

1. If $a^2 = c \times c_2$ and
2. $b^2 = c \times c_1$ then, ΔABC is right angled triangle.

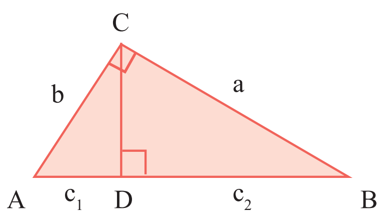


Figure5.19:

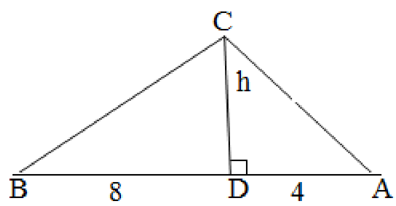
Example 5.7:

In figure 5.22 to the right

AD = 4cm, BD = 8cm

AC = $4\sqrt{3}cm$ and BC = $4\sqrt{6}cm$ and $m\angle ADC = 90^{\circ\circ}$

Is ΔABC a right angled?



Solution:

i. $(AC)^2$? AB × AD

$(4\sqrt{3}\text{cm})^2$? 12cm × 4cm Since AB = AD + BD

$48\text{cm}^2 = 48\text{cm}^2$

Hence $b^2 = c \times c_2$

ii. $(BC)^2$? AB × BD

$(4\sqrt{6}\text{cm})^2$? 12cm × 8cm

$96\text{cm}^2 = 96\text{cm}^2$

Hence $a^2 = c \times c_1 a^2 = c \times c_1$

Therefore, from (i) and (ii), $\Delta ABC \Delta ABC$ is a right angled triangle.

Exercise 5.3.

1. In Figure 5.20 to the right if DB = 8cm and AD = 4cm, then find the lengths of

   a. $\overline{AB}$ c. $\overline{AC}$

   b. b.$\overline{BC}$ d. $\overline{DC}$

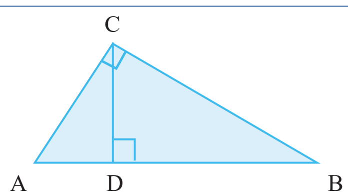


Figure5.20:

2. In Figure 5.21, find the length of x, a and c. If AD = 4cm

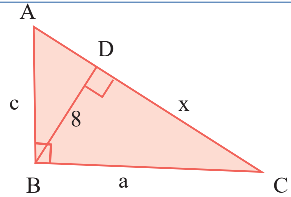


Figure5.21:

3. How long is the altitude of an equilateral triangle if a side of a triangle is

   a. 4cm? b. 8cm?

4. In figure 5.22, AD = 3.2cm,

   DB = 1.8cm, AC= 4cm and

   BC = 3cm. Is ΔABC is a right angled?

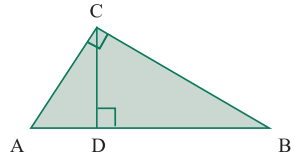


Figure5.22:

5.3.2. The Pythagoras's theorem and its converse

Competencies:

At the end of this section, students should:

- Derive the Pythagoras theorem by using Euclid's theorem and paper folding
- Apply Pythagoras' Theorem and its converse for solving related problem

Pythagoras's Historical note

Pythagoras was a Greek religious leader and a philosopher who made developments in astronomy, mathematics and music theories.

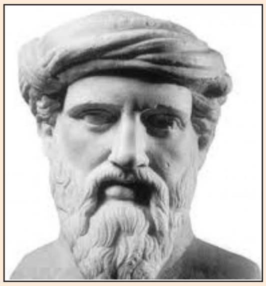


Figure5.23:

Pythagoras theorem was introduced by the Pythagoras. He started a group of Mathematicians who works religiously on numbers and lived like monks.

Finally, the Greek Mathematician stated the theorem hence it is called by his name as the "Pythagoras theorem" (Pythagoras's)

Activity: 5.3.

1. Pythagorean triples consists of three whole numbers $a, b,$ and c which obey the rule: $a^2 + b^2 = c^2$
   a. When a = 15 and b = 20 then find the value of c.
   b. When a = 24 and c = 40 then find the value of b.
2. Pythagorean Theorem states that $a^2 + b^2 = c^2$ for the sides $a, b,$ and c of a right- angled triangle. When a = 5, b = 12 and c = 13. Find three more sets of a rational numbers which satisfy Pythagoras' theorem

One of the most useful theorems in mathematics provides a means for finding the length of any side of a right triangle, given the lengths of the other two sides. The sides are related by the equation

$$(hypotenuse)^2 = (leg1)^2 + (leg2)^2.$$

This relationship is known as the Pythagoras theorem.

Theorem 5.4 **(Pythagoras' Theorem)**

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. That is, if legs of lengths a and b, hypotenuse of length c, then $a^2 + b^2 = c^2$

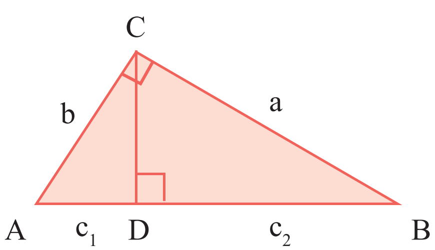


Figure5.24:

Algebraic Method of proof of Pythagoras theorem

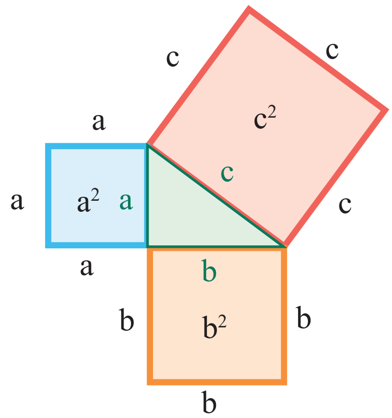


Figure5.25:

Consider the above figure; three squares and one right angled triangle

The area of square with length a $= a^2$

The area of square with length b $= b^2$

The area of square with length c $= c^2$ but the area of square with side c is equal to the square of the length of hypotenuse.

The Pythagoras theorem says that the square of the two smaller squares add up to the same as the area of the largest square.

That is, $a^2+b^2 = c^2$ which is proved.

Algebraic Method proof of Pythagoras theorem will help us in deriving the proof of the Pythagoras theorem by using of a, b and c (values of the measures of the sides corresponding to sides BC, AC, and AB respectively)

Consider four right triangles where b is base, a is altitude and c is hypotenuse. Arrange these four congruent right triangles in the given square, whose side

is $a + b$.

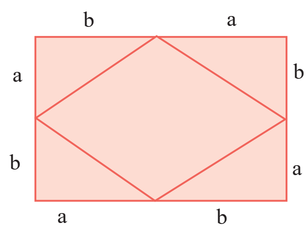


Figure5.26:

The area of the square so formed by arranging the four triangles is c^2.

The area of a square with side $(a+b)$ =$(a+b)$ = Area of 4 triangles + Area of square with side c.

This implies $(a+b)^2 = 4 \times \frac{1}{2}(a \times b) + c^2$ but

$(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$

$a^2 + b^2 + 2ab = 2ab + c^2$

Therefore, $a^2 + b^2 = c^2$

Pythagoras theorem proof using similar triangles

Proof: Let ΔACB be right angled triangle, the right angle at C.

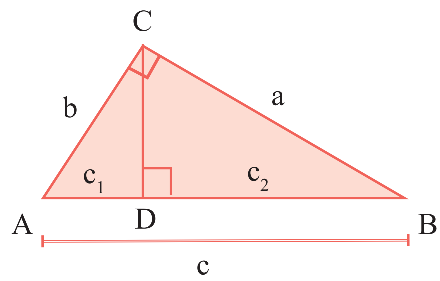


Figure5.27:

Given: ΔACB is a right triangle and $\overline{CD} \perp \overline{AB}$.

We want to show that: $a^2 + b^2 = c^2$

	Statements	**Reasons**
1.	$a^2 = c_2 \times c$	Euclid's Theorem
2.	$b^2 = c_1 \times c$	Euclid's Theorem
3.	$a^2 + b^2 = (c_2 \times c) + (c_1 \times c)$	Sum of step1and step2
4.	$a^2 + b^2 = c(c_2 + c_1)$	Taking c as a common factor
5.	$a^2 + b^2 = c(c)$	$c = c_2 + c_1$
6.	$a^2 + b^2 = c^2$	Proved

Example 5.8:

What is the length of the hypotenuse of a right angle triangle with leg lengths are 3cm and 4cm.

Solution:

Let $a = 3cm$, $b = 4cm$ and c be the length of the hypotenuse. Then

$a^2 + b^2 = c^2$ [Pythagorean theorem]

$(3cm)^2 + (4cm)^2 = c^2$

$9cm^2 + 16cm^2 = c^2$

$25cm^2 = c^2$

$c^2 = \sqrt{25cm^2}$

$c = 5cm$

Therefore, the hypotenuse is $5cm$ long.

Example 5.9:

Consider a right angle triangle. The measure of its hypotenuse is 13cm. One of the sides of a triangle is 5cm.Find measure of the third sides using the Pythagoras theorem formula?

Solution:

Let the unknown leg be b. Then

$(Hypotenuse)^2 = (leg1)^2 + (leg2)^2$

$(13cm)^2 = b^2 + (5cm)^2$

$169cm^2 = b^2 + 25cm^2$

$b^2 = 169cm^2 - 25cm^2$

$b^2 = 144cm^2$

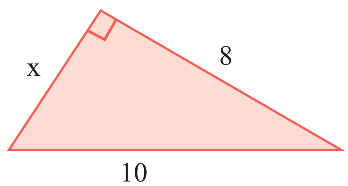
$b = \sqrt{144cm^2}$

$b = 12cm$

Therefore, the base right angled triangle is $12cm$

Example 5.10:

Find the length of the leg of the given right angle triangle



Solution:

Let xx be the length of the required leg. Then

$$(hypotenuse)^2 = (leg1)^2 + (leg2)^2$$

$$(10cm)^2 = (8cm)^2 + (x)^2$$

$$100cm^2 = 64cm^2 + x^2$$

$$x^2 = 100cm^2 - 64cm^2$$

$$x^2 = 36cm^2$$

$$x = \sqrt{36cm^2}$$

$$x = 6cm$$

Therefore, the other leg is $6cm$ long.

Note:

When three positive integers can be the length sides of a right triangle, this set of numbers is called Pythagoras triple. The most common Pythagoras triples is 3, 4, and 5. If we multiply each number of a Pythagorean triples by some positive integer x, then new triples created is also a Pythagorean triples because it satisfy the relation $a^2 + b^2 = c^2$.

In general patterns of $3x, 4x, 5x$ for x is a positive integer forms a Pythagorean triples. There are many other families of Pythagorean triples including 5x 12x, 13x or 8x, 15xx, 17x where x is a positive integer.

Example 5.11:

Let a, b and c be a Pythagorean triple numbers:

a. if $a = 15$ and $b = 20$,then find the value of c

b. if $a = 24$ and $c = 40$,then find the value of b

Solution:

Since a, b and c are a Pythagorean triple numbers

a. $a^2 + b^2 = c^2$

$(15)^2 + (20)^2 = c^2$

$225 + 400 = c^2$

$c^2 = 625$

$c = \sqrt{625}$

$c = 5$

b. $a^2 + b^2 = c^2 \; a^2 + b^2 = c^2$

$(24)^2 + b^2 = (40)^2$

$576 + b^2 = 1600^2$

$b^2 = 1600 - 576$

$c^2 = 1024$

$c = \sqrt{1024}$

$c = 32$

If the lengths of the sides of a triangle are known, then we can determine whether the triangle is a right triangle by applying the converse of Pythagoras Theorem.

Theorem 5.6. (Converse of Pythagoras' Theorem)

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right angled triangle.

Figure5.28:

Consider the following figure.

Given: Let ΔABC in which $(AC)^2 = (AB)^2 + (BC)^2$

To prove: $\angle B = 90°$

Construction: Draw a right angle triangle PQR in which $\angle Q = 90°$ and PQ =AB, BC=RQ

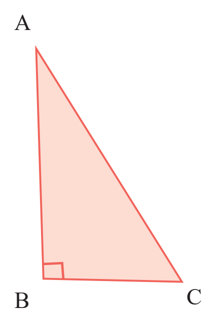


Figure5.29:

Proof: In a right angle triangle PQR

$(PR)^2 = (PQ)^2 + (RQ)^2$ [Pythagoras Theorem]   Figure 5.29

Since, $PQ = AB$, $RQ = BC$

$(PR)^2 = (AB)^2 + (BC)^2$ But $(AC)^2 = (AB)^2 + (BC)^2$ So, $PR = AC$

In ΔABC and ΔPQR we have $AB = PQ AB = PQ, BC = QR$, and $AC = PR$

So, $\Delta ABC \cong \Delta PQR$ by SSS.

Thus $\angle B = \angle Q$. But $\angle Q = 90°$ So $\angle B = 90°$.

Example 5.12:

Determine whether the triangle whose sides have lengths 11, 60, and 61 is a right triangle.

Solution:

We want to show that

$a^2 + b^2 = c^2$. $(11)^2 + (60)^2$? $(61)^2$

121 + 3600 ? 3721

3721 = 3721 Hence, the triangle is a right triangle.

Exercise 5.4.

1. State the Pythagoras theorem in your own words.
2. Find the lengths of the sides of a rhombus whose diagonals are 6cm and 8cm.
3. Find the unknown sides length of each of the following figures

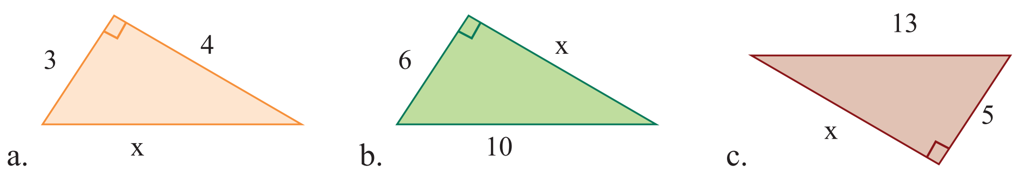


Figure5.30:

4. What is the length of the hypotenuse of a right angle triangle with leg lengths of 27cm and 36cm?

5. If diagonal of a square is 12cm long, then how long is each sides of a square?
6. A 15m ladder lean against the side of the house, and the base of the ladder is 9m away. How high above the ground is the top of the ladder?
7. A man travels 24km due west and then 10km due north. How far is the man now from the starting place?
8. In the figure given below, find the length of

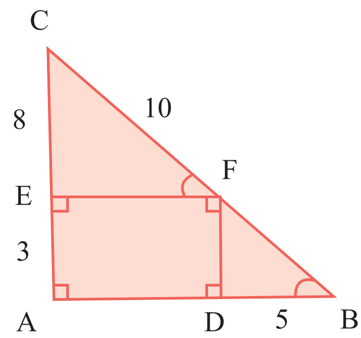


Figure5.31:

a. AE
b. DF
c. EF
d. AB
e. Is ΔCEF is a right- angled?
f. Is ΔADF is a right- angled?

UNIT SUMMARY

1. The sum of the degree measures of the interior angles of a triangle is equal to 180°
2. The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.
3. **Theorem (Euclid's Theorem)**

   In a right angled triangle with an altitude to the hypotenuse, the square of the length of each leg of a triangle is equal to the product of the hypotenuse and the length of the adjacent segment into which the altitude divides the hypotenuse:

Symbolically

i. $(BC)^2 = AB \times BD$
or $a^2 = c \times c_2$

ii. $(AC)^2 = AB \times AD$
or $b^2 = c \times c_1$

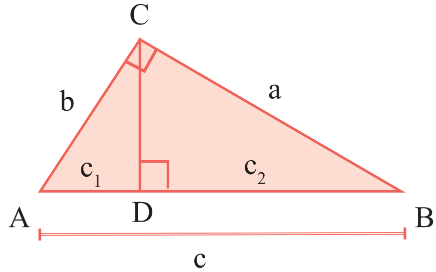


Figure5.32:

4. Theorem (converse of Euclid's Theorem)

In a triangle, if a square of each shorter side of a triangle is equal to the product of the length of the longest side of the triangle and the adjacent segment into which the altitude to the longest sides divides this side, then the triangle is right angled.

Symbolically:

i. If $a^2 = c \times c_2$ and
ii. $b^2 = c \times c_1$, then ΔABC is right angled triangle

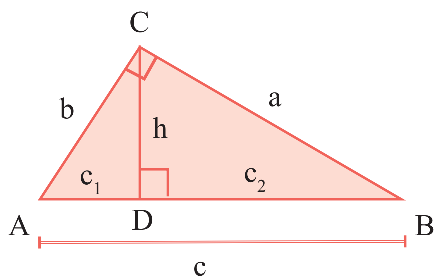


Figure5.33:

5. Theorem (Pythagoras' Theorem)

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of other two sides. That is legs of lengths a and b hypotenuse of length c, then $a^2 + b^2 = c^2$

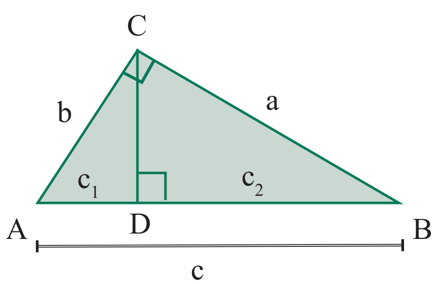


Figure5.34:

6. When three positive integers can be the length of the sides of a right triangle, this set of numbers is called Pythagoras triple. The most common Pythagoras triple is 3, 4, and 5

7. **Theorem 5.3.2 (Converse of Pythagoras' Theorem)**

   In a triangle, if square of one side is equal to the sum of the square of the other two sides, then angle opposite the first side is a right angle

REVIEW EXERCISE

1. If the measure of the angles of a triangle are $6x, 8x$ and $10x$, then give the measures of each angle.
2. One acute angle of a right triangle measure 37°. Find the measures of the other acute angle?
3. Prove that the sum of the degree measures of the interior angles of a triangle is equal to 180°.
4. What is the value of x in figure below?

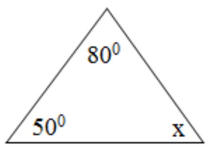


Figure5.35:

5. What is the measure of angle B in the figure below?

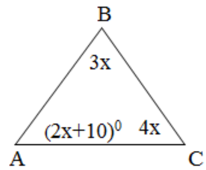


Figure5.36:

6. Calculate the measure of angle A in the figure below

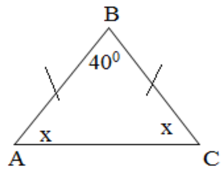


Figure5.37:

7. Prove that the measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

8. Given that for a triangle the two interior angles $(x + 10)°$ and $(x + 20)°$ are remote to an exterior angle $(6x - 30)°$, then find the value of x?

9. Calculate the values of x and y in the following triangle.

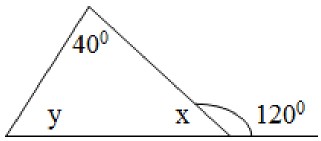


Figure5.38:

10. State Pythagoras Theorem

11. State Euclid's Theorem

12. In figure 5.39 to the right if DB = 8cm and AD = 4cm

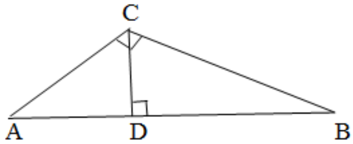


Figure5.39:

Then find the lengths of

a. $\overline{AB}$ b. $\overline{BC}$

b. $\overline{AC}$ d. $\overline{DC}$

13. A triangle has sides of lengths 16cm, 48cm and 50cm respectively. Is the triangle a right angled triangle?

14. In figure 5.40 below. What is the value of x?

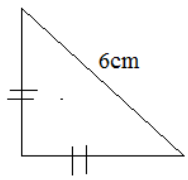


Figure5.40:

15. A tree 18 meter-high is broken off 3 meter from the ground. How far from the foot of the tree will the top strike the ground?

16. A rectangle has its sides 5cm and 12cm long. What is the length of its diagonal?

17. What is the length of the hypotenuse of a right angle triangle with leg lengths are 9cm and 12cm?

Unit 6

6. CIRCLES

Learning Outcomes:

At the end of this unit, learners will able to:

- Have a better understanding of circles.
- Realize the relationship between lines and circles.
- Apply basic facts about central and inscribed angles and angles formed by intersecting chords to compute their measures.
- Apply real-life situations in solving geometric problems.

Main content

6.1. Circles

6.2. Applications of circle

- Summary
- Review Exercise

Introduction

In lower grades especially in grade 6 mathematics you had learnt about circles and its parts like center and radius. In this unit we consider the concepts of chord, circumference of a circle, classification of arcs, sector of a circle, segment of a circle, central and inscribed angle and positional relations between a line and a circle. Of all simple geometric figures, a circle is perhaps the most appealing. Have you ever considered how useful a circle is? Without circles there would be no watches, wagon, automobiles, electricity or many other modern conveniences.

6.1. Circles

Group work 6.1

Discuss with your partner.

i. Take a compass and set a fixed point on the paper.

ii. Draw a closed figure. Then answer the following questions.

a. The small puncture hole (the impression that the metal compass point makes on the paper is called ________

b. The compass setting distance is called _______

6.1.1. Lines and circles

Competencies:

At the end of this section, students should:

- Identify the different types of arcs, sectors, segments.
- Describe the concepts "tangent" and "secant" of a circle.

Activity: 6.1.

1. Name any three parts of a circle.
2. The diameter of a circle is 18cm. what is the radius of the circle? What do you conclude?
3. How are chords and secants of circles alike? How are they different?

Definition 6.1

A circle is a set of points in a plane having the same distance from a fixed point.

- The fixed point, O is called the center of the circle.
- The distance between the center of the circle and any point of the circle, r is called the radius(plural: radii) of the circle.

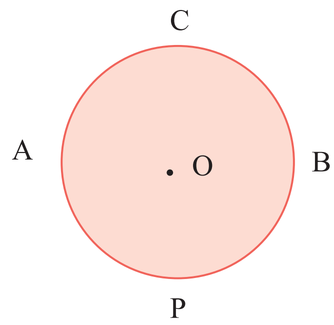


Figure 6.1:

Note:

- A capital letter is used to designate the center of the circle. If the center of a circle is designated by the letter O, the circle is referred to as circle O.

Several basic terms with which you should be familiar with are associated with circles. These terms are defined below and illustrated in Figure 6.2

Definition 6.2

a. A chord of a circle is a line segment joining two points of the circle. Thus, in the figure, $\overline{AB}$ is a chord.

b. A diameter of a circle is a chord through the center. Thus, $\overline{CD}$ is a diameter of circle O.

c. Secant of a circle is a line that intersects the circle at two points. Thus, $\overleftrightarrow{EF}$ is a secant.

d. A tangent of a circle is a line that touches the circle at one and only one point. Thus, $\overleftrightarrow{GH}$ is a tangent to the circle at P. The point P is called the point of tangency or point of contact.

e. The distance around a circle is called the circumference of the circle.

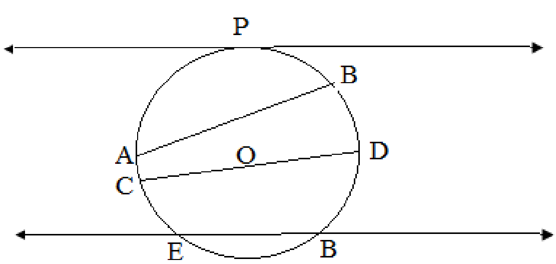


Figure 6.2:

Note:

- All radii of a circle are congruent.
- A diameter of a circle consists of two radii that lie on the same straight line

Example 6.1:

In circle O, radius OA = $3x - 10$ and radius OB = $x + 2$. Find the length of a diameter of circle O.

Solution:

Since all radii of a circle are congruent, OA = OB

$$3x - 10 = x + 2$$

$$3x = x + 12$$

$$x = 6$$

$$OA = OB = x + 2 = 6 + 2 = 8$$

Therefore, the length of a diameter of circle O is 2 x 8 = 16.

Activity: 6.2.

Draw a circle and a line intersecting the circle at

a. two points
b. one point
c. no points

what do you conclude about the relation between a line and a circle?

Note:

- There is a positional relationship between a line and a circle. A line may intersect a circle at no points, one point, or two points.
- Consider circle O and point P, point Q and point B. Since the length of $\overline{OP}$ is less than the radius, then point P is called an interior point and the length of $\overline{OQ}$ is greater than the radius, then point Q is an exterior point.

Point B is on the circle (OB = r)

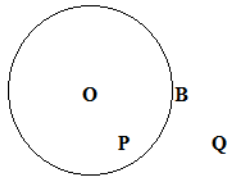


Figure 6.3:

Exercise 6.1.

1. Write true if the statement is correct and false if it is not.
   a. A diameter divides a circle into two congruent parts.
   b. The point at which a tangent line intersects the circle to which it is tangent is the point of tangency.
   c. A straight line can intersect a circle in more than two points.
   d. A diameter of a circle is twice the radius.
   e. The portion of the secant within the circle is necessarily the chord.
2. Tell whether the line or line segment is best described as a chord, a secant, a tangent, or radius of circle O.
   a. $\overline{AD}$ b. $\overline{HB}$ c. $\overline{OD}$ d. $\overleftrightarrow{EG}$

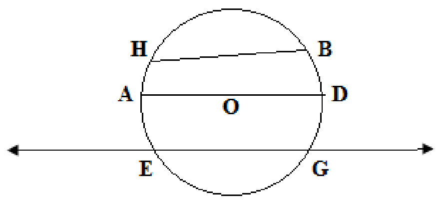


Figure 6.4:

Arcs and its Type

Definition 6.3

An arc of a circle is a part of the circumference of a circle. The symbol for an arc is ⌒.Thus, $\overset{\frown}{AB}$ stands for arc AB..

In the figure 6.5, A, B and C are points on circle O and ∠ AOB intersects the circle at two distinct points, A and B, separating the circle in to two arcs.

3. If $m\angle$ AOB < 180^0, points A and B and the points of the circle in the interior of ∠ AOB make up minor arc AB, written as AB.
4. Points A and B and the points of the circle not in the interior of ∠ AOB make up major arc AB. Usually a major arc is named by three points: The two end points and any other point on the major arc. Thus, the major arc with end points A and B is written as ACB.
5. A semi-circle is an arc of a circle whose end points are the end points of a diameter of the circle; its measure is 180^0.

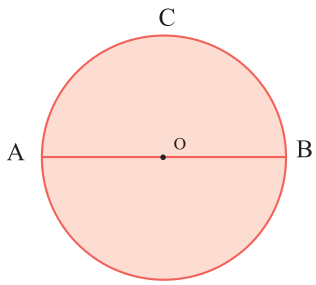


Figure 6.5:

Note:	A chord of a circle subtends the arcs which it cut off on the circle. The arc of a circle cut off by a chord is subtended by the chord. In figure 6.6, arc ADB is subtended by chord $\overline{AB}$.

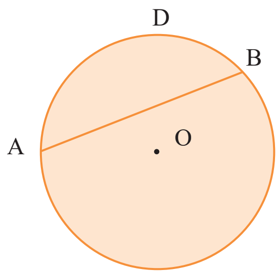


Figure 6.6:

Definition 6.4

A sector of a circle is the part of a circle bounded by two radii and their intercepted arc.

In the figure, region AOB is called a minor sector and region ADB is called a major sector.

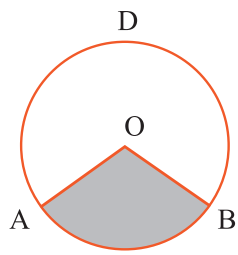


Figure 6.7:

Definition 6.5

A segment of a circle is the part of a circle bounded by a chord and its subtended arc.

Regions ABC and ADB are called segments. Region ABC is the minor segment and region ADB is called the major segment.

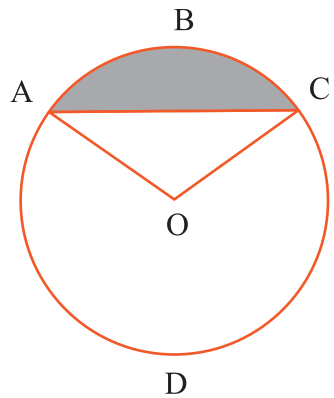


Figure 6.8:

Exercise 6.2.

1. Identify a chord, a secant, a tangent, point of tangency, a sector and a segment for the following figure

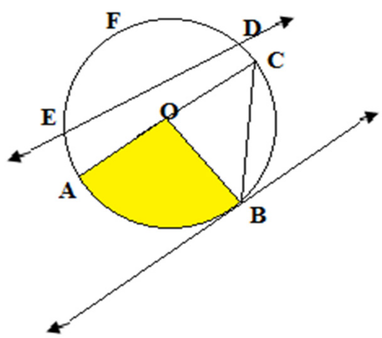


Figure 6.9:

2. Write true if the statement is correct and false if it not.
   a. a secant of a circle contains chord of the circle.
   b. tangent is a segment whose end points is on a circle.
   c. if point A is an interior point of circle O, then it is possible to draw a tangent line that contains point A.
   d. Sector is a region bounded by two radii and the intercepted arc.

3. Consider figure 6.10 and describe the relationship between lines and circle O, points and circle O.

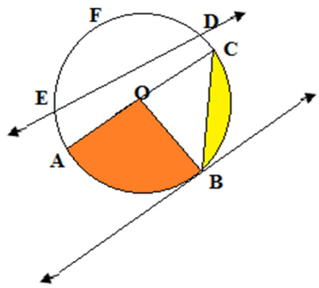


Figure 6.10:

4. Consider figure 6.11 and answer the following questions.
   a. Name four minor arcs of circle O.
   b. Name four different major arcs of circle O.
   c. Name major arc TES in three different ways.

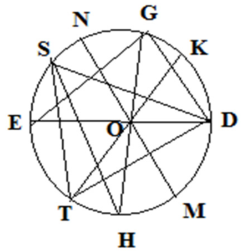


Figure 6.11:

6.1.2. Central angle and inscribed angle

Competencies:

At the end of this section, students should:

- Describe the central angles and inscribed angles.
- Find the measure of central angle or inscribed angle or the intercepted arc based on the given information.

Activity: 6.3.

A and C are points on a circle with center O. Draw a point B on the circle so that $\overline{AB}$ is a diameter.

1. What angle in your diagram is:
   a. an inscribed angle? b. a central angle?
2. What is the intercepted arc of
   a. $\angle ABC$ b. $\angle AOC$

We now extend the discussion to angles whose vertices lie at the center called central angle and whose vertices lie on the circle called inscribed angle.

Definition 6.6

A central angle is an angle whose vertex is the center of the circle and whose sides are radii of the circle.

In figure 6.12, ∠AOB is a central angle. The reflex angle AOB is called the reflex central angle.

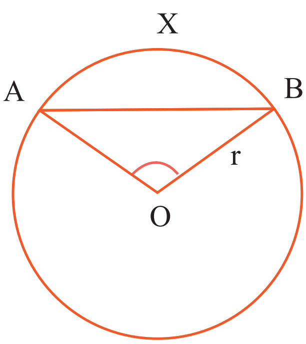


Figure 6.12:

Note:	From figure 6.12, Arc AXB is said to be intercepted by ∠AOB and ∠AOB is said to be subtended by arc AXB.

A central angle is measured by its intercepted arc. That is, the measure of an arc of a circle may also be related to an angle whose vertex is the center of the circle and whose sides intercept arcs of the circle. In figure 6.13, ∠AOB is called a central angle. The minor arc that lies in the interior of the angle is defined to have the same measure as its central angle, ∠AOB.

The measure of the central angle ∠AOB is equal to the measure of the intercepted arc AXB. That is, $m\angle AOB = m\angle AXB$.

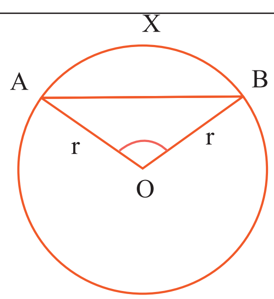


Figure 6.13:

Example 6.2:

For the following figure, if $m\angle AOB = 67^0$ and $\overline{CB}$ is a diameter of circle O, then find

a. $m(\overset{\frown}{AB})$

b. $m(\overset{\frown}{AC})$

c. $m(\overset{\frown}{ADB})$

d. $m(\overset{\frown}{BDC})$

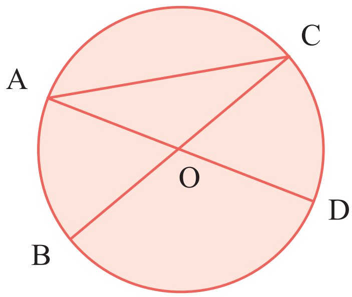


Figure 6.14:

Solution:

a. $m(\overset{\frown}{AB}) = m\angle AOB = 67^0$

b. $m(\overset{\frown}{AC}) = m\angle AOC = 180^0 - 67^0 = 113^0$

c. $m(\overset{\frown}{ADB}) = 360^0 - m AB = 360^0 - 67^0 = 293^0$

d. $m(\overset{\frown}{BDC}) = 180^0$

Let us now consider the situation in which the vertex of the angle is a point on the circle and the sides of the angle are chords of the circle.

Definition 6.7

An inscribed angle is an angle with its vertex on the circle and whose sides contain chords of the circle.

In the figure 6.15, $\angle ABC$ is inscribed angle for circle O.

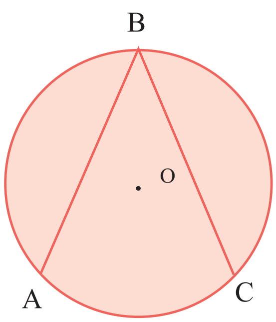


Figure 6.15:

Note:	The arc formed by the intersection of the two sides of the angle and the circle is called an intercepted arc. From figure 6.15, $\angle ABC$ is an inscribed angle and arc AC is an intercepted arc. We say that $\angle ABC$ is inscribed in the arc ABC and $\angle ABC$ is subtended by arc APC.

You know the measure of an arc is equal to the measure of the corresponding central angle, then how do the measures of inscribed angle ABC and its intercepted arc AC compare?

Theorem 6.1.

The measure of an inscribed angle is one-half of the measure of its intercepted arc.

In the figure, $m\angle ABC = \frac{1}{2} m\ \text{arc}AC$

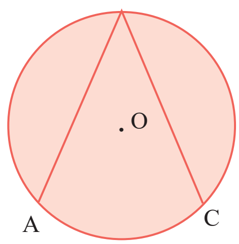


Figure 6.16:

Example 6.3:

Let $\overline{AC}$ be the diameter of circle O and $m\angle AOB = 112^0$ as shown in the figure. Then find $m\angle OAB$.

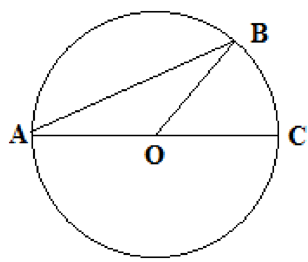


Figure 6.17:

Solution:

$m\angle AOB = mAB$. Hence, $mAB = 112^0$

$m\text{BC} = mABC - mAB$

$= 180^0 - 112^0 = 68^0$

$$m\angle OAB = \frac{1}{2} mBC$$

$$= \frac{1}{2}.68^0 \quad = 34^0$$

Example 6.4:

If $m\angle ABC = 75^0$ and $mAD = 60^0$ as shown in the figure, then find mDC.

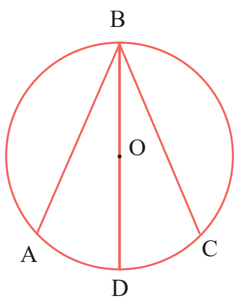


Figure 6.18:

Solution:

$$m\angle ABC = \frac{1}{2} M(\overset{\frown}{ADC})$$

$$75^0 = \frac{1}{2} M(\overset{\frown}{ADC})$$

$$M(\overset{\frown}{ADC}) = 150^0. \text{ But } m(\overset{\frown}{DC}) = M(\overset{\frown}{ADC}) - M(\overset{\frown}{AD})$$

$$M(\overset{\frown}{AD}) = 150^0 - 60^0$$

$$= 90^0$$

Example 6.5:

ΔABC whose vertices lie on a circle so that its sides divide the circle into arcs whose measures have the ratio of 2: 3: 7. Find the measure of the smallest angle of the triangle.

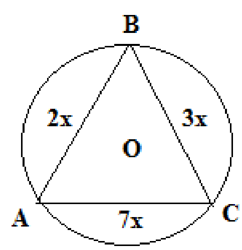


Figure 6.19:

Solution:

First, determine the measures of the arcs of the circle.

$$mAB + mBC + mAC = 360^0$$

$$2x + 7x + 3x = 360^0$$

$12x = 360^0$

$x = 30^0$

The measures of the arcs of the circle are:

$M(\overset{\frown}{AB}) = 2x = 60^0$

$M(\overset{\frown}{BC}) = 7x = 210^0$

$M(\overset{\frown}{AC}) = 3x = 90^0$

Each angle of the triangle is an inscribed angle. The smallest angle lies opposite the arc having the smallest measure.

Hence, $m\angle C = \frac{1}{2} m(\overset{\frown}{AB}) = \frac{1}{2} \times 60^2 = 30^0$

Theorem 6.2.

In a circle, inscribed angles subtended by the same arc are congruent.

That is $m\angle ABE = m\angle ACE$

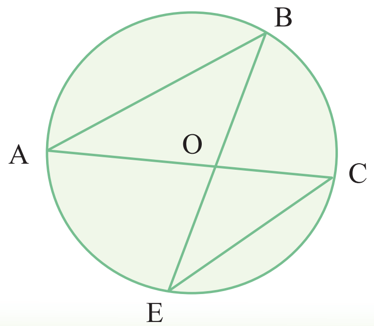


Figure 6.20:

Example 6.6:

In figure 6.20 if $m\angle ACE = 65^0$, then find $m\angle ABE$.

Solution:

Since, ∠ACE is an inscribed angle and its measure is one- half of AE ,

then $M(\overset{\frown}{AE}) = 2 \times 65^0 = 130^0$.

Similarly, ∠ABE is an inscribed angle and its measure is one- half of

($\widehat{AE}$) , then $m\angle ABE = \frac{1}{2} \times 130^0 = 65^0$.

Or by the above theorem, ∠ACE and ∠ABE intercept the same arc, then $m\angle ABE = m\angle ACE = 65^0$

Exercise 6.3.

1. Write true if the statement is correct and false if it is not.
   a. If the measure of the central angle is halved, then the length of the intercepted arc is doubled.
   b. An angle inscribed in a semi-circle is a right angle.
   c. Angles inscribed in the same arcs are congruent.
   d. A central angle is measured by its intercepted arc.
   e. A central angle of a circle is an angle whose vertex is on a circle and whose sides contain chords of the circle.
   f. The measure of the central angle is twice of the measure of the inscribed angle.
2. A circle is divided into three arcs in the ratio 3:2:5. The points of division are joined to form a triangle. Find the largest angle of the triangle.
3. In the fig. 6.21 O is the centre of the circle. Find
   a. m(∠BAD) and m(∠CBA)
   b. m(∠ADC) and m(∠ABC)
   $m(\widehat{AB})$, $m(\widehat{CD})$ and m(∠CBD)

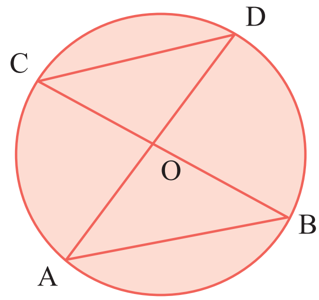


Figure 6.21:

4. If $\overline{EC}$ is the diameter of the given circle in figure 6.22 and $m(\widehat{BAE}) = 100^0$ and $m(\widehat{AE}) = 60^0$,find the degree measure of angle x and y.

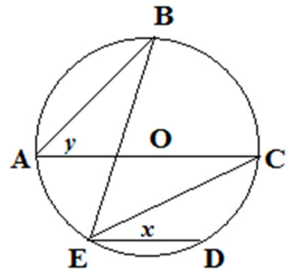


Figure 6.22:

Group work 6.2

Discuss in groups and describe what you notice.

i. Construct a circle O with any radius.
ii. Draw a chord, and label it $\overline{CD}$.
iii. Construct the perpendicular bisector of $\overline{CD}$.
iv. What do you notice about the perpendicular bisector of $\overline{CD}$?
v. Draw another chord, and label it $\overline{AB}$.
vi. Construct a line that passes through the center O and the midpoint of chord $\overline{AB}$.
vii. What do you notice about the line and the chord?
viii. Draw another chord $\overline{PQ}$.
ix. Construct a line perpendicular to $\overline{PQ}$ and passes through the center of the circle.
x. What do you notice the line and the chord?

Conclusion:

Form group work 6.2, you observe the following facts.

- The perpendicular bisector of a chord passes through the center of the circle.
- A line which passes through the center and bisects a chord is perpendicular to the chord.
- A line through the center of a circle perpendicular to a chord bisects the chord.

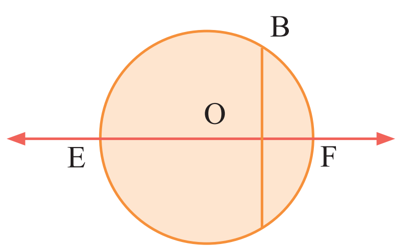


Figure 6.23:

6.1.3. Angles formed by two intersecting chords

Competencies:

At the end of this sub-topic, students should:

- Describe the angle formed by two intersecting chords.
- Solve problems related to angle formed by two intersecting chords inside a circle.

You know that the measure of an angle inscribed in a circle is one-half of the measure of the intercepted arc. In this sub-topic you will learn about the measure of an angle formed by two intersecting chords within a circle.

Activity: 6.4.

In figure 6.24, if $m(\overset{\frown}{AD}) = 72^0$ and $m(\overset{\frown}{BC}) = 48^0$, then find the value of $\angle$APD

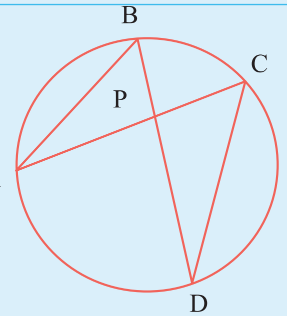


Figure 6.24:

Theorem 6.2.

The measure of an angle formed by two chords intersecting inside a circle is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

$$m\angle APD = \frac{1}{2}[m(\overset{\frown}{AXD}) + m(\overset{\frown}{BYC})]$$

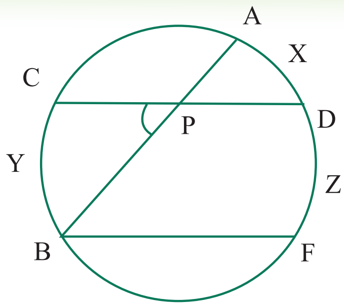


Figure 6.25:

Example 6.7:

In figure 6.26 shown, O is the center, $m\angle ABD = 30^0$ and $m$arcCD = 60^0. Then find

a. $m\angle CPB$

b. $m\angle BDK$

c. $m\angle AQK$

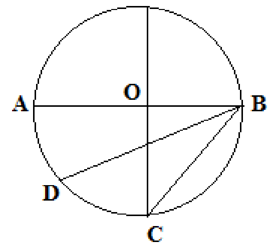


Figure 6.26:

Solution:

$m\angle ABD = 30^0$. Then $m(\overset{\frown}{AD}) = 2\, m\angle ABD = 2 \times 30^0 = 60^0$

Since $\overline{AB}$ is a diameter, $m(\overset{\frown}{AD}) + m(\overset{\frown}{DC}) + m$arcCB $= 180^0$

$m(\overset{\frown}{CB}) = 180^0 - 120^0 = 60^0$

a. $m\angle CPB = \frac{1}{2} m(\overset{\frown}{CB}) + m$arcAD)

$= \frac{1}{2}(60^0 + 60^0)$

$= 60^0$

b. $\overline{DK}$ is a diameter, then $m(\overset{\frown}{DC}) + m(\overset{\frown}{CB}) + m(\overset{\frown}{BK}) = 180^0$

$m(\overset{\frown}{BK}) = 180^0 - 120^0 = 60^0$

$m\angle BDK = \frac{1}{2} m\, BK$

$= \frac{1}{2} 60^0 \quad = 30^0$

c. $m(\overset{\frown}{AK}) = 180^0 - 60^0 = 120^0$

$m\angle AQK = \frac{1}{2}[m(\overset{\frown}{DC}) + m(\overset{\frown}{AK})]$

$= \frac{1}{2}(60^0 + 120^0)$

$= 90^0$

Example 6.8:

An angle formed by two chords intersecting within a circle is 54^0, and one of the intercepted arcs measures 40^0. Find the measures of the other intercepted arc.

Solution:

Consider figure 6.26.

$$m\angle BPC = m(\widehat{BC}) + m(\widehat{AD})$$

$$54^0 = \frac{1}{2}40^0 + \frac{1}{2}m(\widehat{AD})$$

$$108^0 = 40^0 + m(\widehat{AD})$$

$$m(\widehat{AD}) = 68^0$$

Example 6.9: Prove that the measure of an angle formed by two chords intersecting in the interior of a circle is equal to one-half the sum of the measures of the two intercepted arcs.

Proof:

Consider circle O with chords $\overline{AC}$ and $\overline{BD}$ intersect at point P.(refer figure 6.26)

$\angle$APD is an exterior angle of ΔAPB.

Hence, $m\angle APD = m\angle A + m\angle B$

$$\frac{1}{2}m(\widehat{BC}) + \frac{1}{2}m(\widehat{AD})$$

$$\frac{1}{2}m(\widehat{BC}) + \frac{1}{2}m(\widehat{AD})$$

Exercise 6.4.

1. Write true if the statement is correct and false if it is not.
   a. A diameter perpendicular to a chord bisects the chord.
   b. A perpendicular bisector of a chord passes through the center of the circle.
   c. The measure of the angle formed by two chords of a circle is equal to the sum of the measure of the arc subtending the angle and its vertically opposite angle.
2. In the figure 6.27, $m\angle APB = 110^0$ and $m\angle BAC = 40^0$.Then
   a. Find the sum of $m(\widehat{AD})$ and $m(\widehat{BC})$.
   b. $m\angle ACD$

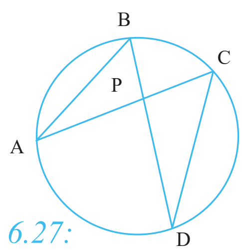


Figure 6.27:

3. In the figure 6.28, $m\angle CBD = 45^0$, $m\angle ABD = 40^0$ and $m\text{arc}AB = 90^0$. Then find $m\angle APD$

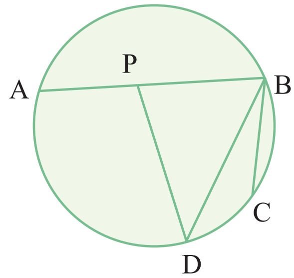


Figure 6.28:

6.2. Applications of Circle

Competency:

At the end of this sub-topic, students should:

- Solve the application problems.

Circles are present in real life, both in the nature world such as Satellite's orbit around the earth, Mushrooms with domed caps and in manmade creations such as camera lenses,coins, cups, rings, wheel, roundabout,a sniper target grid etc. In this section we will discuss some real life applications of circles using examples.

Example 6.9:

Which part of a circle can be found in a do not smoke sign when the line crosses the image of cigarette?

Figure 6.29:

Solution: Diameter.

Example 6.10:

Dawit observes that the time is 5.00 hrs. What is the angle between the hands of the clock?

Solution:

The small hand on a clock shows the hours on the face of a clock. So, if the hour hand lies on 6.00 hrs, then the angle between the hands is 180^0.

$6hr = 180^0$.

$1hr = \ ?x$

$$x = \frac{1hr \times 180^0}{6hr} = 30^0$$

Thus, 1hr measures 30^0. Therefore, a 5 hr hand measures $5 \times 30^0 = 150^0$

SUMMARY UNIT

1. A circle is the set of all points in a plane having the same distance from a fixed point. The fixed point is called the center and the fixed distance is the radius of the circle
2. A chord is a line segment whose end points lie on the circle.
3. A diameter is a chord that passes through the center of the circle.
4. A tangent line is a line that intersects a circle in exactly one point. The point of contact is called the point of tangency.
5. A secant line is a line that intersects a circle in two different points.
6. Every secant line includes a chord of the circle.
7. An arc is a curved portion of a circle.
8. A chord determines an arc of a circle. So, we say that the chord intercepts the arc.
9. Arcs are classified in the following ways:
   a. Semi-circle: an arc whose end points are also end points of a diameter of a circle.
   b. Minor arc: the part of a circle less than a semi-circle.
   c. Major arc: the part of a circle greater than a semi-circle.
10. A sector of a circle is the region bounded by two radii and an arc of the circle.
11. A segment of a circle is the region bounded by a chord and the arc of the circle.

12. A central angle is an angle whose vertex is at the center of thecircle.
13. An inscribed angle is an angle whose vertex is on the a circle and whose sides contain chords of the circle.
14. To prove a chord is a diameter show that the chord divides the circle into congruent (equal) arcs.
15. The measure of an inscribed angle is equal to one-half of the measure of its intercepted arc.
16. The measure of an angle formed by two chords intersecting inside a circle is one – half of the sum of the measure of the arcs subtending the angle and its vertical opposite angle.

REVIEW EXERCISE

1. Write true if the statement is correct and false if it not.
   a. Minor arc is the part of a circle greater than a semi-circle.
   b. Inscribed angles intercepted by the same arc are congruent.
   c. A central angle is not measured by its intercepted arc.
   d. A circle O has a radius of 6cm. Then the length of the longest chord of circle O is 12cm.
   e. An angle is inscribed in a circle and intercepts an arc with measures 108^0, then the measure of the inscribed angle is 54^0.
   f. 180^0 cannot be the measure of an inscribed angle.
2. In the figure below, if $m\angle ABD = 45^0$, m arcBC = 100^0. Then

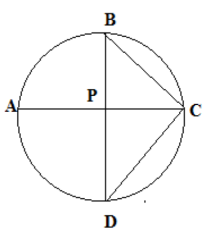


Figure 6.30:

   i. Find
   a. $m\angle ACD$
   b. $m\angle BAC$
   c. $m\angle BPC$
   ii. Is the point P lies at the center of the circle? Explain.
   iii. What type of triangle is formed by $\overline{CD}$, $\overline{PD}$ and $\overline{PC}$?

3. In the figure below, O is the center of the circle. If $m\angle POS = 95^0$, then find

   a. $m\angle QSR$

   b. $m\angle PQS$

   c. $m\angle QPR$

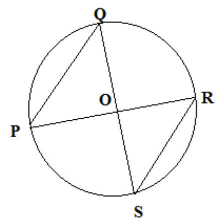


Figure 6.31:

4. In figure 6.32, $m\angle AEC = 70^0$, $m\angle ADB = 25^0$, and $m\angle BFC = 80^0$. Find

   a. $m\angle DAE$

   b. $m(\widehat{DE})$

   c. $m(\widehat{BC})$

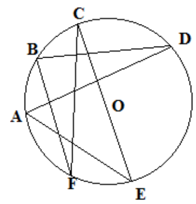


Figure 6.32:

5. In figure 6.33, $m\angle BCD = 90^0$, $m\angle BAC = 49^0$ and $m\angle ADB = 61^0$. Find

   a. $m\angle ACB$

   b. $m\angle CAD$

   c. $m\angle ABC$

   d. $m\angle BEC$

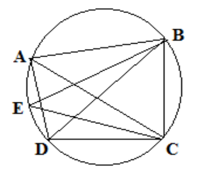


Figure 6.33:

6. In figure 6.34 below $\overline{PT}$ is a chord and O is the center of the circle and $m\angle PBT = 70^0$, then calculate

   a) $m\angle TPO$

   b) $m$arcTQ

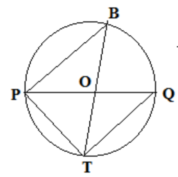


Figure 6.34:

7. In figure 6.35 below, $\overline{AD}$ is a diameter and $m(\widehat{AC}) = 132^0$. Find

   a. $m\angle OCD$

   b. $m$arcCD

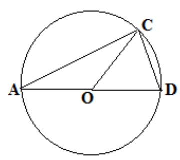


Figure 6.35:

8. In the figure below, what is the measure of the inscribed angle?

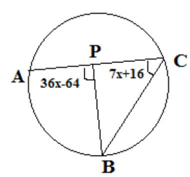


Figure 6.36:

Unit 7

7. SOLID FIGURES AND MEASUREMENTS

Learning Outcomes:

At the end of this unit, learners will able to:

- Identify parts of solid figures.
- Find the surface area of solid figures.
- Find the volume of solid figures.
- Solve applications problems of solid figures and measure-ments.

Main content

Introduction

In plane geometry, by definition, every point and line of each figure lie in a single plane; in contrast, the figure of solid do not lie in a single plane. In this unit, we extend our geometry from the plane into three dimensional solid figures. We want to define some solid figures, develop some formulas about the surface area of solids, and volume of solids. The concept of the area of a plane surface is easily extended to surface area of a solid flat. Each and everything around us is a solid shape. For instance laptop, phone, or tablet are a solid figures. Similarly, the chair, the bench, etc. are also solid figures. Look around and see what different shapes are around you? Are all three dimensional? Here, we have only covered the basic shapes.

7.1. Solid Figures

Competencies:

At the end this sub-unit. Students should:

- Identify parts of a prisms, cylinders, pyramid and cones
- Name different types of prisms, cylinders, pyramids and cones based on their bases.

Solid figures are three-dimensional objects. What this means is that solid figures have a width, a length, and a height. For instance, computer, laptop, phoneetc. has a width, a length, and a height.

In mathematics, there are many solid figures. Most of them are prisms, Cylinders, pyramids and Cones.

7.1.1. Prisms and Cylinders

We encounter prisms and cylinders everywhere; most boxes and most rooms are rectangular prisms, most cans are cylinders. When we find out how much cardboard there is in the box, when we need the area of the walls to paint in a room, or when we need to find how much tin is needed to make a can, we are finding the **surface area**

of prisms and **cylinders**. When we find out how much milk is in the container, how much soup is in the can, and how much chocolate is in the packet, we are finding the **volume of prisms** and **cylinders**?

Group work 7.1

A B C D E

Figure 7.1:

1. Name the above figures as prisms, cylinders, cones and pyramids.
2. List some solid objects in your surroundings.
3. Can you give model examples of Prisms and Cylinders?
4. Define Prism and cylinder in your own words.
5. Identify some parts of prisms and cylinders.
6. How many vertices, edges and faces dotriangular prisms and cylinders have?
7. Explain why a cube is a rectangular prism.
8. Name the following prisms

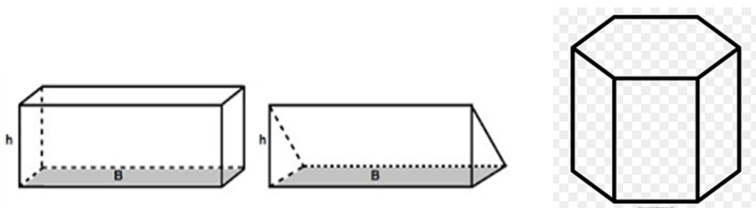


Figure 7.2:

1. Prisms

Definition: 7.1:

A prism is athree dimensional figure in which two of the faces, called the bases of the prism, are congruent polygons in parallel plane.

Depending on the shape of its base a prism can be triangular prism, rectangular prism, pentagonal prism, hexagonal prism and so on. For instance,

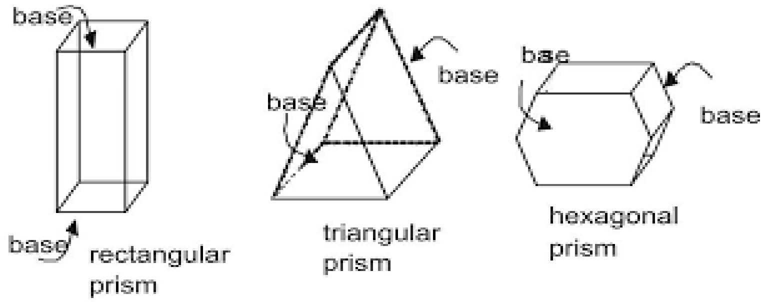


Figure 7.3:

Consider the rectangular prism as shown below

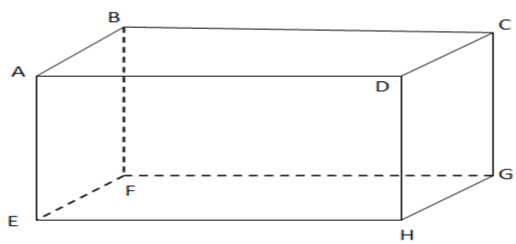


Figure 7.4:

- The vertices of the rectangular prism are A, B, C, D, E, F, G, and H
- A prism has two bases: upper base and lower base. The rectangular region ABCD is the upper base and rectangular region EFGH is the lower base.
- The line segments;$\overline{AB}$,$\overline{BC}$,$\overline{CD}$,$\overline{DA}$,$\overline{EF}$,$\overline{FG}$,$\overline{GH}$, and $\overline{HE}$ are edges of the bases.

- The line segments; $\overline{AE}$, $\overline{BF}$, $\overline{CG}$ and $\overline{DH}$ (the segments that connect the vertices) are the lateral edges of the prism.
- The rectangles ABFE, BCGF, CDHG, and ADHE (the surfaces between corresponding sides of the bases) are called the lateral faces of the prism.
- An altitude of a prism is a line segment perpendicular to each of the bases with an end point on each base. The height of a prism is the length of an altitude.

Definition 7.2.

A right prism is a prism in which the lateral sides are all perpendicular to the bases.

Note:	Since the bases are parallel, the corresponding sides of the bases are congruent, parallel line segments. Therefore, each lateral face has a pair of congruent, parallel sides, the corresponding edge of the bases, and is thus parallelograms. Therefore, the lateral edges of a prism are congruent and parallel. All of the lateral sides of a right prism are rectangles.

Example7.1:

The bases of a prism are equilateral triangles. The length of one edge of a base is 4cm and its height is 5cm.

a. How many lateral edges does this prism have?

b. What is the shape of the lateral face?

Solution:

a. Because this is a prism with a triangular base, the prism has three lateral edges.

b. Because it is a right prism, the lateral sides are rectangles.

2. Cylinders

Definition7.3:

A cylinder is a solid figure with congruent non-polygonal bases that lie in parallel planes. If the bases are circular, the cylinder is circular cylinder.

Note:

- The line passing through the centers of the two circular bases is called the axis of the cylinder.
- The altitude of a cylinder is the perpendicular distance between its bases.
- The radius of the base is also called the radius of the cylinder.
- A cylinder is called a right cylinder if the segment joining the centers of the bases is perpendicular to the bases.

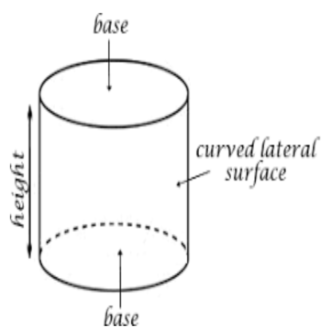


Figure 7.5:

Exercise 7.1.

1. Write true if the statement is correct and false otherwise.
   a. The intersection of the faces of a prism are the edges of the prism
   b. A line through the centers of the bases of a circular cylinder is perpendicular to the diameter of the bases.
   c. The intersection of the edges of a prism are the vertices of the prism.
   d. The upper and the lower bases of circular cylinder are circles of equal radii.
   e. A triangular prism has a triangular lateral face.
   f. The bases of a prism lie on parallel planes.
   g. A cylinder is a circular prism.
2. Describe the differences between a prism and a cylinder. Describe their similarities.

3. Give a mathematical name of the solids given below.
   a. Soup can b. Shoe box
4. Make a sketch of the described solid
   a. Right rectangular prism with a 5cm square base and a height of 6cm.
   b. Right cylinder with a diameter of 14cm and a height of 9cm.
5. How many edges, faces and vertices does
   a. a cylinder has?
   b. a rectangular prism has?

7.1.2. Pyramids and Cones

The main focus of this sub-unit is to name the vertex, lateral face, base, and altitudes of the given pyramids and cones.

Activity: 7.1.

1. Define pyramids and cones in your own words.
2. Can you give examples of pyramids and cones in your surroundings.
3. Answer the following questions based on the given figures
   a. Name the vertices of the pyramids and the cones.
   b. Name the base of the pyramid.
   c. Name the lateral faces of the pyramid.
   d. Name the altitude of the cone.

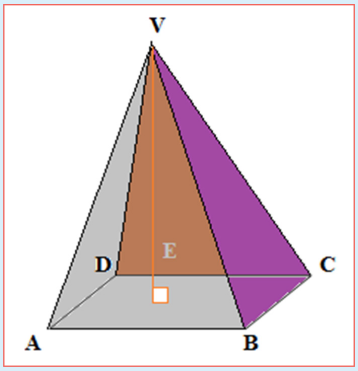


Figure 7.6:

1.Pyramids

Definition 7.4:

A pyramid is a polygon in which the base is a polygon and the lateral faces are triangles with a common vertex.

From the figure 7.7 we define the following terminologies.

- The polygonal region ABCD is called the base of the pyramid.
- The point V outside of the plane of the base is called the vertex of the pyramid.
- The triangles VAB, VBC, VCD, and VDA are called lateral faces of the pyramid.
- The intersection of two lateral faces is called lateral edges. Thus, $\overline{VA}$, $\overline{VB}$, $\overline{VC}$, and $\overline{VD}$ are lateral edges of the pyramids.
- The intersection of the base and a lateral face is a base edge. Thus, $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ are the edges of the base of the pyramid.
- The altitude of the pyramid is the perpendicular distance between the base and the vertex.

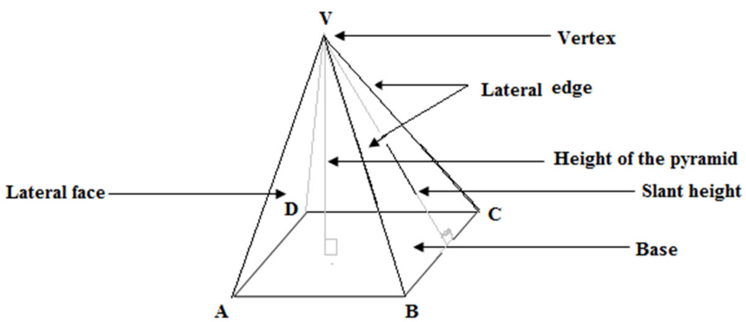


Figure 7.7:

2. Cones

Cones can be found in a variety of things we see every day such as funnel, ice cream, the birthday hat etc. In this section we will define the vertex, lateral face, base and altitude of the given cone.

Definition 7.5:

A circular cone is a solid figure formed by joining all points of a circle to a point not on the plane of the circle.

- The point outside the plane and at which the segments from the circular region joined is called the vertex of the cone.
- The flat surface, the circle, is called the base of the cone and the curved closed surface is called the lateral face of the cone.
- The perpendicular distance from the base to the vertex is called the altitude of the cone.
- A circular cone with the foot of its altitude is at the center of the base is a right circular cone.

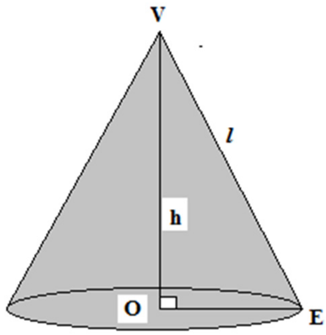


Figure 7.8:

Exercise 7.2.

1. Write true if the statement is correct and false if it not.
   a. Cone is a pyramid.
   b. A cone has a flat surface and a curved surface.
   c. A triangular pyramid has three vertices and three faces.
   d. A cone has one vertex and one curved edge.
   e. A pyramid cannot have any parallel faces.
2. Describe the differences between pyramids and cones. Describe their similarities.
3. Can a pyramid have rectangles for a lateral faces? Explain.
4. Sketch the described solid.
   a. A right pyramid that has a triangular base with a base edge of 4cm and a height of 7cm.
   b. A right cone that has a radius of 3cm and a height of 5cm.

7.2. Surface Area and Volume of Solid Figures

7.2.1. Surface area of Prisms and Cylinders

Competency:

At the end of this section, students should:

- Find the surface area of prisms and cylinders

In this sub-section you will become more acquainted with the these familiar geometric figures and you will learn how to compute their surface area in a systematic way.

Group work 7.2

1. Draw the nets of each of the following figures.

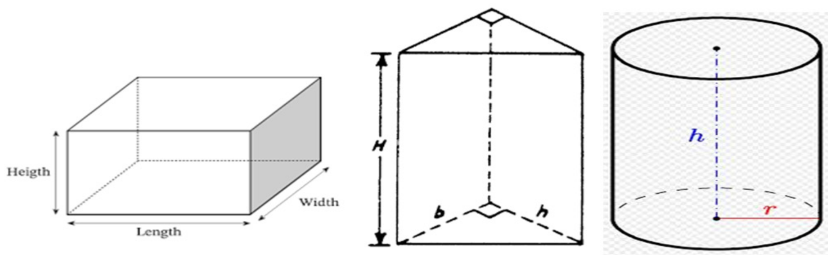


Figure 7.9:

2. Sketch the box that results after the net has been folded. Use the shaded face as a base.

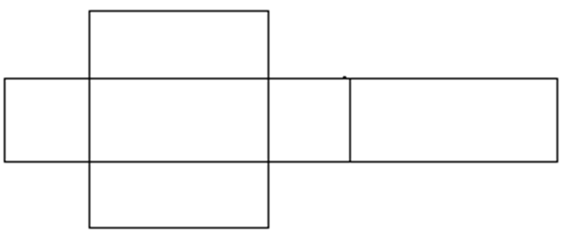


Figure 7.10:

The net of a three dimensional shape is a two dimensional shape that can be folded to form a three dimensional shape or a solid. Any solid figure can have multiple nets.

Definition 7.6:

A net is a pattern of shapes on a piece of paper that are arranged so that the net can be folded to make a hollow surface.

Example 7.2:

a. The net of a cylinder looks like a rectangle with two circles attached at opposite ends.

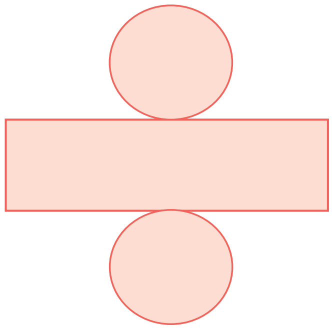


Figure 7.11:

b. The net of a triangular prism consists of two triangles and three rectangles. The triangles are the bases of the prism and the rectangles are the lateral faces.

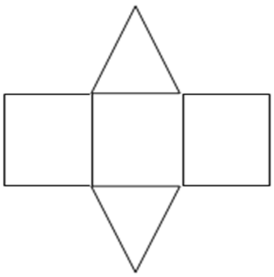


Figure 7.12:

A. Surface area of Prisms.

There are two types of surface areas: **Lateral surface area** and **Total surface area**.

The lateral surface area, denoted by A_L of the prism is the sum of the areas of all lateral faces of the prism.

Example7.3:

Derive a formula for the lateral surface area of a rectangular prism using its net.

Solution:

The surface area of a right rectangular prism can be calculated by representing the

3D figure into a 2D net; to make the shapes easier to see. After expanding the 3D shape into 2D shape we will get six rectangles (See figure 7.12). Pair wise (Front and back, right and left, top and bottom) rectangles have equal size.

Thus, area of front face = area of back face = lh

area of left face = area of right face = wh

area of top face = area of bottom face = lw

Since, the lateral surface area is the sum of the areas of all lateral faces, then

A_L= = area of front face+ area of back face + area of left face+ area of right face

$= lh + lh + wh + wh$

$= 2lh + 2wh$

$= 2h(l + w)$

$= ph$ where $p = 2(l + w)$ is the perimeter of the base.

The total surface area, denoted by A_T is the sum of the areas of all the faces.

Total surface area (A_T) = A_L + area of top face + area of bottom face

$= ph + lw + lw$

$= ph + 2lw$

$= A_L + 2A_B$ where $A_B = 2lw$ is the area of the base.

Example7.4:

Derive a formula for the lateral surface area of a rectangular prism using its 3D shape.

Solution:

Refer the 3D shape of a rectangular prism (figure 7.12). It has six faces: Two bases and four lateral faces. Then,

Lateral Surface Area = the sum of the area of the four lateral faces.

A_L = a(ABGH) + a(ADEH) + a(DCFH) + a(BCFG)

$= wh + lh + wh + lh$

$= 2lh + lh + 2whwh$

$= h\,2(l + w = ph$

$A_T = A_L +$ areas of the two bases

$= A_L + a(ABCD) + a(EFGH)$

$= A_L + 2lw$

$= A_L + 2A_B$

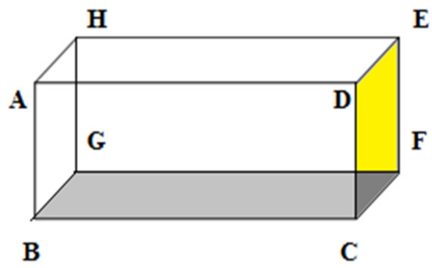


Figure 7.13:

In general, the lateral surface area of a prism is $A_L = ph$ where p is the perimeter of the base.

The total surface area of a prism is $A_T = A_L + 2A_B$ where A_B is the area of the base.

Group work 7.3

Using the above technique derive a formula for surface area of right triangular prism.

Example 7.4:

The base of a rectangular prism is ABCD, where AB = 6cm, BC = 3cm and height h = 4cm. Then find

a. Lateral surface area of the prism
b. Total surface area of the prism

Solution:

$A_L = ph$

$= 2(6 + 3) \times 4 = 72\ cm^2.$

$A_T = A_L + 2A_B$

$= 72 + 2(6 \times 3)$

$= 108 cm^2$

Example 7.5:

The base of a triangular prism is ΔABC, where AB = 6cm, BC = 8cm and $m\angle B = 90^0$. If the height h of the prism is 7cm. Find

a. The lateral surface area

b. The total surface area

Solution:

a. First determine the length of AC.

Using Pythagoras theorem,

$AC^2 = BC^2 + AB^2$

$= 8^2 + 6^2$

AC = 10. Then,

$A_L = ph$

$= (6 + 8 + 10) \times 7$

$= 168cm^2$

b. $A_T = A_L + 2A_B$

$= 168 + 2 (\frac{11}{22} AB. BC)$

$= 168 + 6 \times 8$

$= 216 \text{ cm}^2$

B) Surface area of cylinders

To derive a formula to the surface area of a circular cylinder, consider the net of the cylinder. It has two congruent and parallel circular bases and a rectangular face of length $2\pi r$ and width h as shown in figure 7.14. Therefore,

The lateral surface area (A_L) = area of the rectangle

$= 2\pi r \times h$

$A_L = 2\pi rh$

Similarly, the total surface area $(A_T) = A_L + 2$. area of the bases

$$A_T = A_L + 2\pi r^2$$

$$A_T = 2\pi rh + 2\pi r^2$$

$$A_T = 2\pi r\,(h + r)$$

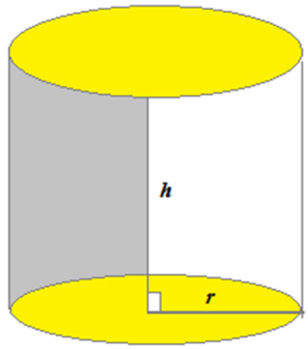


Figure 7.14:

In general,

The lateral surface area of a circular cylinder is $A_L = 2\pi rh$ and

The total surface area of a circular cylinder is $A_T = 2\pi r(h + r)$

Example 7.7:

Find the total surface area of a right circular cylinder whose radius is 8cm and height is 12cm.

Solution:

$$A_T = 2\pi r\,(h + r)$$

$$= 2\pi \times 8 \times (12 + 8)$$

$$A_T = 320\pi \text{ cm}^2$$

Example 7.8:

Find the height of the right circular cylinder if its lateral surface area is $108\pi\pi \text{cm}^2$ and radius of the cylinder is 6cm.

Solution:

$$A_L = 2\pi rh$$

$$108\pi \text{cm}^2 = 2\pi.\,6.\,h = 12\pi h$$

$h = \frac{108\pi}{12\pi} = 9\text{cm}$

Example 7.9:

The total surface area of a right circular cylinder of radius 7cm is $112\pi\text{cm}^2$. Then find the height of the cylinder.

Solution:

$A_T = 2\pi r\,(h + r)$

$112\pi = 2\pi \times 7\,(h + 7)$

$\frac{112\pi}{14\pi} = h + 7$

$8 = h + 7$

$h = 1\text{cm}.$

Therefore, the height of the cylinder is 1cm.

Example 7.10:

The sum of the height and the radius of a right circular cylinder is 9cm. If the surface area of the cylinder is $54\,\pi\text{cm}^2$, then find the height of the cylinder.

Solution:

Let h be the height and r be the radius of the cylinder.

$h + r = 9$

$A_T = 2\pi r\,(h + r)$

$54\,\pi = 2\pi\,(9 - h)9$

$\frac{54\pi}{2\pi} = 81 - 9h$

$27 = 81 - 9h$

$h = 6cm$

Therefore, the height of the cylinder is 6cm.

Exercise 7.3.

1. Write true if the statement is correct and false if it is not
   a. The total surface area of a triangular prism can be calculated by adding the areas of 5 faces.
   b. A 3D object with two parallel and congruent circular bases is a cylinder.
   c. Surface area is measured in cubic units.
   d. The dimensions of the rectangular prism are increased 2 times, and then the surface area will increase 8 times.
   e. In a cylinder, if the radius is doubled and height is halved, then its lateral surface area will be the same.
   f. The two-dimensional representation of all of the faces is a net.
2. Find the lateral surface area of a triangular prism whose height is 10cm and the dimensions of each of its bases are 3cm, 6cm, and 7cm.
3. Find the lateral surface area of the prism if the perimeter of the base is 100cm and its height is 5cm. Also, find the total surface area of the same prism if its base area is $50cm^2$.
4. The lateral surface area of a right circular cylinder of height 8cm is $88\pi cm^2$. Then find the diameter of the base of the cylinder.
5. The radius of two similar right circular cylinders is 3cm and 12cm. find the ratio of their altitudes.

7.2.2. Volume of Prisms and Cylinders

Competency:

At the end of this sub- unit, students should:

- Find the volume of prisms and cylinders.

In the previous sub-topic you have learnt about how to compute the surface area of prisms and cylinders. In this sub-topic, you will learn how to compute the volume of prisms and cylinders.

Activity: 7.2.

1. Define volume of a solid figure in your own words.
2. Heran said that if two solids have equal volume and equal heights, then they must have congruent bases. Do you agree with Saron? Justify your answer.
3. Find the volume of the prism if its base area is $48cm^2$ and its height 18cm.
4. The volume of a right circular cylinder is 784π cm^3 and the height of the cylinder is 16cm. what is the radius of the cylinder?

As area is thought of as the number of square units that can be compute within a two dimensional boundary, the volume of a solid can be the number of cubic units that will fit within the boundary of a three-dimensional solid.

Definition 7.7

The volume of a solid is the number of cubic units contained in its interior. That is, the amount of space it occupies. Volume is measured in cubic units such as m^3, cm^3 etc.

A) Volume of Prism

The volume V of a rectangular prism is equal to the product of its length (l), width (w) and height (h). That is, $V = l \times w \times h$. By using this relationship, special formula can be derived for determining the volume of a cube. That is,

$V = l \times l \times l = l^3$.

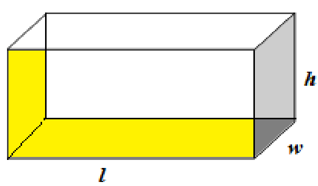
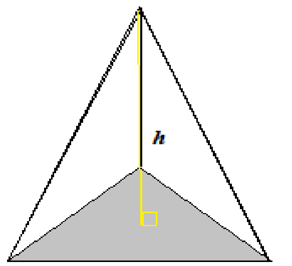


Figure 7.15:

The volume V of a right triangular prism equals the product of its base area A_B and its height h. That is, $V = A_B \times h$.

The volume V of a prism is $V = A_B h$, where A_B is the area of the base and h is the height. *Figure 7.16:*	

Example 7.10

Determine the height of a rectangular prism that has a base dimension of 8cm, 6cm and a volume of $312cm^3$.

Solution:

Let $l = 8$cm and $w = 6$cm.

$$V = l \times w \times h$$

$$312cm^3 = 8 \times 6 \times h$$

$$h = \frac{312}{48}$$

$$h = 6.5 \text{ cm}$$

Therefore, the height of the prism is 6.5cm

Example 7.11:

The bases of a right prism is ΔABC with D is a point on $\overline{CD}$, $\overline{AB}$ is perpendicular to $\overline{BC}$. If AB = 10cm, Ac = 10cm, BC= 12cm, AD = 8cm and EB = 15cm. find the volume of the prism.

Solution:

Since this is a right prism, all of the lateral faces are rectangles and the height of the prism, EB, is the height of each face.

Each base is an isosceles triangle. The length of the base of the isosceles triangle is BC = 12cm, and the length of the altitude to the base of the triangle is AD = 8cm.

Area of ΔABC $A_B = \frac{1}{2}.BC. AD$

$= \frac{1}{2}.12.\ 8$

$= 48\ cm^2$

Since the prism is a right prism, the height of the prism is BE = 15cm.

Volume of the prism $V = A_B \cdot h$

$= 48 \times 15$

$= 720\ cm^3$

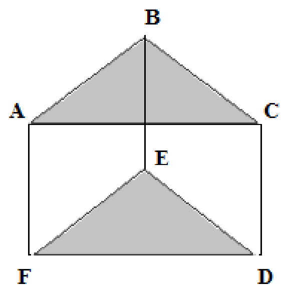


Figure 7.17:

Example 7.12:

The base of a prism is an equilateral triangle each of whose sides measures 4cm. If the altitude of the prism measures 5cm, then find the volume of the prism.

Solution:

Since the base is an equilateral triangle, first determine the height, say y of the base.

$y^2 + 2^2 = 4^2$

$y^2 = 16 - 4$

$y = \sqrt{12}$. So,

$A_B = \frac{1}{2}.\ AB.\ y$

$= \frac{1}{2}.\ 4.\ \sqrt{12} \quad = 2\sqrt{12}$

Then, $V = A_B.\ h$

$V = 2\sqrt{12}\ .\ 5$

$= 10\sqrt{12}\ cm^3$ Figure 7.18

B. Volume of cylinder

Group work 7.4

Discuss with your groups: Take two sheets of paper that measure 12cm by 28cm and form two cylinders, one with the height as 12cm and one with the height as 28cm. Do the cylinders have the same volume? Explain.

The formula for the volume of the cylinder can be derived easily because a cylinder can be thought of as a prism with a circular base. The volume of a prism is $V = A_B h$ since the base is a circle then the base area A_B is the area of a circle,

$A = \pi r^2$. By substitution, the volume of a cylinder can be found by using the formula $V = \pi r^2 h$.

The volume V of a cylinder is $V = = \pi r^2 h$

Example 7.13:

A right circular cylinder of height 12cm has a volume of 972π cm^3. Find the radius of the base of the cylinder.

Solution:

$$V = \pi r^2 h$$

$$972\pi = \pi r^2 . 12$$

$$\frac{972\pi}{12\pi} = r^2$$

$$r^2 = 81$$

$$r = 9$$

Therefore, the radius of the base is 9cm.

Example 7.14:

Consider a circular cylinder. If a new cylinder is formed by doubled the radius and by halved the height, then what is the volume the new cylinder?

Solution:

Let V_1 be the volume of the first cylinder.

Let r_1 and h_1 be the respective radius and height of the first cylinder.

Since, $r_2 = 2r_1$, $h_2 = \frac{1}{2}h_1$, then

$V_2 = \pi r_2^2 h_2$

$= \pi\,(2r_1)^2.\frac{1}{2}h_1$

$= 4.\frac{1}{2}.\pi\, r_1^{\,2}.h_1$

$= 2\pi\, r_1^{\,2}.h_1$

$= 2V_1$

Therefore, the volume of the new cylinder becomes twice that of the original cylinder.

Exercise 7.4.

1. Write true if the statement is correct and false if it is not.
   a. The volume of a cylinder with base diameter d and height h is given by $V = \frac{\pi d^2 h}{4}$.
   b. If the volume of a rectangular prism whose dimensions l, $\frac{4}{l}$, and h is 1cm^3, then the value of h is $\frac{1}{4}$.
   c. Measuring the space region enclosed by the solid figure is the volume of the solid figure.
   d. If two solid figures are congruent, then they have the same volume.
   e. Measuring the surface constituting the solid is the area of the solid figure.
2. The volume of a triangular prism is 204cm^3. If its height is 17cm, then find the area of its base.
3. Given the length, width, and height of the rectangular prism as 8cm, 5cm, and 16cm respectively. Find the volume of the rectangular prism.
4. Find the volume of a circular cylinder if its diameter is 12cm and its height is 20cm.

7.3. Applications on Solid Figures and Measurements

Competency:

At the end of this sub-unit, students should:

- Solve applications problems of solid figures and measurements.

You recall that

Volume	Surface Area
✶ Measures space inside	✶ Measures outside surface
✶ Includes only space needed to fill inside	✶ Includes all faces
✶ Measured in cubic units	✶ Can be measured using a net

This section focuses on the applications of solid figures.

Example 7.15:

A truck that delivers gasoline has a circular cylindrical storage space. The diameter of the bases of the cylinder is 34cm and the height of the cylinder is 46cm. How many gallons of gasoline does the truck hold?

(Use $1cm^3 = 0.000264$ gal)

Solution:

$V = \pi r^2 h$

$= \pi . 17^2 . 46 \quad = 13,294\ cm^3$

$1cm^3 = 0.000264$ gal

$13,294\ cm^3 = ?$

$V = 13,294 \times 0.000264$ gal

$V = 3.51$ gal

Therefore, the truck holds 3.51 gallons of gasoline.

Example 7.16:

The walls, floor, and ceilings of a room form a rectangular solid. The total surface area of the room is 992 m^2. The dimensions of the floor are 12m by 20m.

a. What is the lateral surface area of the room?

b. What is the height of the room?

Solution:

a. Since the solid is a rectangular prism, then

$$A_T = A_L + 2A_B$$

$$A_T = A_L + 2 \,.\, l.w$$

$$992 = A_L + 2.\ 20.\ 12$$

$$A_L = 992m^2 - 480m^2$$

$$A_L = 512m^2$$

Therefore, the area of the walls of the room is 512m^2

b. Since the solid is a rectangular prism, then

$$A_L = ph = 2(l + w)h$$

$$512m^2 = 2(20 + 12)\ m.\ h$$

$$h = \frac{512}{64}$$

$$h = 8m$$

Therefore, the height of the room is 8m.

Example 7.17:

A fish tank in the form of a rectangular solid is to accommodate 6fish, and each fish needs at least 7500 cm^3 of space. The dimensions of the base are to be 30cm by 60cm. What is the minimum height that the tank needs to be?

Solution:

The minimum volume occupied by 6 fish is 6 x 7500 = 45000cm^3

Since the tank is in the form of a rectangular solid,

$V = l \times w \times h$

$l \times w \times h \geq 45000\ cm^3$

$30cm \times 60cm \times h \geq 45000\ cm^3$

$h \geq \frac{45000}{1800}$

$h \geq 25$

Therefore, the minimum height of the tank would be 25 cm.

Example 7.18:

A gift has a dimensions of 20cm by 35cm by 4cm. you have wrapping paper with dimensions of 43cm by 50cm. Do you have enough wrapping paper to wrap the gift?

Solution:

Total Surface Area of the gift $= 2 \times 20 \times 35 + 2 \times 20 \times 4 + 2 \times 35 \times 4$

$= 1840\ cm^2$

Area of wrapping paper $= 43\ cm \times 50cm$ [rectangular surface]

$= 2150\ cm^3$

Therefore, there is an enough wrapping paper to wrap the gift.

Example 7.19:

Saron wants to build a right circular cylinder out of a card board with bases that have a radius of 6cm and a height of 14cm.

How many square centimeters of card board are needed for the cylinder?

What will be the volume of the cylinder?

Solution:

The amount of materials needed implies the total surface area of the cylinder. Since a cylinder has two bases and one curved surface,

$$A_T = 2\pi rh + 2\pi r^2$$
$$= 2\pi.6.14 + 2\pi.6^2$$

$= 168\pi + 72\pi$

$= 20\pi$ cm^2

Therefore, she needs a 20π cm^2 card board to build a cylinder

a. $V = \pi r^2 h$

$= \pi 6^2 . 14$

$= 504\pi \text{cm}^3$

Therefore, the volume of the constructed cylinder is $504\pi\text{cm}^3$

Example 7.20:

Cereal Company wants to change the shape of its cereal box in order to attract the attention of shoppers. The original cereal box has dimensions of 8cm by 3cm by 11cm. The new box the cereal company is thinking of would have dimensions of 10cm by 10cm by 3cm.

b. Which box holds more cereal?

c. Which box requires more material to make?

Solution:

a. Volume of original box $= l \times w \times h$

$= 8 \times 3 \times 11$

$== 264$ cm^3

Volume of new box $= l \times w \times h$

$= 10 \times 10 \times 3$

$= 300$ cm^3

Therefore, the new box holds more cereal than the original box.

a) Surface Area of original box $= 2 \times 8 \times 3 + 2 \times 8 \times 11 + 2 \times 3 \times 11$

$= 48 + 176 + 66 = 290\text{cm}^2$

Surface Area of new box $= 2 \times 10 \times 10 + 2 \times 10 \times 3 + 2 \times 10 \times 3$

$= 200 + 60 + 60$

$= 320$ cm^3

Therefore, the new box requires more material than the original box.

Example 7.21:

The pedestal on which a statue is raised is a rectangular concrete solid measuring 4m long, 4m wide and 3m high. How much is the cost of the concrete in the pedestal, if concrete costs Birr 320 per m^3

Solution:

First we need to find the volume of the pedestal, which is a rectangular solid.

$$V = l \times w \times h$$

$$= 4 \times 4 \times 3$$

$$= 48 \text{ m}^3$$

$$1\text{m}^3 = \text{Birr } 320$$

$$48 \text{ m}^3 = x$$

$$x = \frac{48\, m^3 \times \; Birr\ 320}{1m^3}$$

$$x = \text{Birr } 15{,}360$$

Therefore, the cost of the concrete is Birr 15, 360.

Exercise 7.5.

1. Find the volume and surface area of a can of soda if the radius is 6cm and the height is 11cm.
2. A cans goods company manufactures a cylindrical can of height 10cm and radius 4cm.
   a. Find the surface area of the can
   b. Find the volume of a can whose radius and height are twice that of the given can.
3. A food company stores food item A in a cylindrical can that is 14 cm tall and has a diameter of 10cm. A company stores food item B in a rectangular can of dimensions 12cm by 13 cm by 5cm. Which can accommodate more food?

UNIT SUMMARY

1. A cylinder whose segments are perpendicular to the bases is called a right cylinder.
2. A cylinder whose bases are enclosed by circles is called circular cylinder.
3. If the base of the cylinder is polygonal region, then the solid is called a prism.
4. A prism whose bases are enclosed by triangles is a triangular prism.
5. A prism whose bases are enclosed by rectangles is a rectangular prism.
6. The perpendicular distance between the bases of a prism is the altitude of the prism.
7. The sum of the areas of lateral faces is called lateral surface area and the sum of the lateral surface area and area of the bases is the total surface area.
8. If the lateral edges of the prism are perpendicular to the bases, then the prism is a right prism.
9. The lateral edges of a prism are congruent and parallel.
10. A cube is a three dimensional solid object bounded by six square faces.
11. A net is a pattern of shapes on a piece of paper that are arranged so that the net can be folded to make a hollow surface
12. Measuring the surface constituting the solid is called the surface area of the solid figure.
13. The lateral surface area of a right prism is equal to the product of the height and the perimeter of the base. That is, $A_L = ph$ where p is base perimeter and h is the height of the prism.

14. The lateral surface area of a circular cylinder is $A_L = 2\pi rh$ and
15. The total surface area of a circular cylinder is $A_T = 2\pi r(h + r)$
16. The volume of a solid is the number of cubic units contained in its interior. That is, the amount of space it occupies. Volume is measured in cubic units such as m^3, cm^3 etc.
17. The volume V of a prism is $V = A_B h$, where A_B is the area of the base and h is the height.
18. A cylinder has two bases that are congruent circles lying in parallel planes. Like a volume of a prism, a solid that also has two congruent bases, the volume of a cylinder is given by the formula $V = A_B h$. In this formula, A_B represents the area of one of the two identical circles which is πr^2.
19. A pyramid is a polygon in which the base is a polygon and the lateral faces are triangles with a common vertex
20. A circular cone is a solid figure formed by joining all points of a circle to a point not on the plane of the circle.

7.2. REVIEW EXERCISE

1. Write true if the statement is correct and false if it is not.
   a. A cube has 6 faces, 12 edges and 8 vertices.
   b. A cone has one circular face, one vertex and has no edges.
   c. A prism always has 2 parallel faces.
   d. The net of a rectangular prism is made from 4rectangles.
   e. A triangle is a possible base of a prism.
   f. A pyramid can have a circular base.
2. How many edges has
   a. a squared pyramid?
   b. a cylinder?
   c. a triangular prism?
   d. a cube?

3. Given that the surface area of a rectangular solid is the sum of the area of its six faces. Then

   a. What is the expected type of solid?

   b. Which type of area is referred?

4. A right prism has bases that are squares. The area of one base is $81cm^2$. The lateral surface area of the prism is $144cm^2$. What is the length of the altitude of the prism?

5. The bases of a prism are right triangles whose edges measure 9cm, 40cm and 41cm. The lateral sides of the prism are perpendicular to the bases. The height of the prism is 15cm.

   a. What is the shape of the lateral sides of the prism?

   b. What are the dimensions of each lateral sides of the prism?

   c. What is the total surface area of the prism?

6. If the total surface area of a right circular cylinder is $884\pi cm^2$ and its radius is 2cm, then what is the length of the altitude?

7. The sum of the height and radius of a right circular cylinder is 9cm. if the total surface area is $81\pi cm^2$, then what is the radius of the cylinder?

8. The length, width and height of a rectangular solid are in the ratio of 3: 2: 1. If the volume of the box is $138cm^3$, what is the total surface area of the box?

9. The dimensions of a box are 16cm by 11cm by 9cm. a small cubical box measures 2cm long each side. How many of these small boxes fit into the bigger box?

10. The volume of a right circular cylinder is $252cm^3$ and the radius of the base is 4cm. Find is the height of the cylinder.

11. The areas of the bases of a cylinder are each $124cm^2$ and the volume of the cylinder is $116\pi cm^3$. Find the height of the cylinder.

12. Arega built a wooden, cubic toy box for his son. Each side of the box measures 13cm.

    a. How many square centimeter of wood did he used to build the box?

    b. How many cubic centimeters of toys will the box hold?

Unit 8

8. INTRODUCTION TO PROBABILITY

Learning Outcomes:

At the end of this unit, learners will able to:

- Understand the concept of probability.
- Find event, sample space and probability of simple events.
- Apply problems of real-life situations in solving the probabilities.

Main content

8.1. The concept of Probability

8.2. Probability of Simple events

8.3. Applications of Probability

- Summary
- Review Exercise

Introduction

Group work 8.1

1. Discuss what result is expected for the following experiments.
   a. A ball is dropped from a certain height.
   b. A spoon full of sugar is added to a cup of milk.
   c. Petrol is poured over fire.
   d. Tossing a coin.
   e. Throwing a die.
   f. Drawing a card from a pack of cards.
2. The weather forecaster announces that the probability of heavy rain with thunder in the afternoon is $\frac{1}{4}$. What does it mean? Explain.

In each of the first three experiments, the result or outcome is certain, and known in advance. That is, in experiment (a), the ball is certain to touch the earth and in (b) the sugar will certainly dissolve in milk and in (c) the petrol is sure to burn. But in the experiments d – e the results are uncertain. For example, when a coin is tossed everyone knows that there are two possible outcomes namely head and tail but no one could say with certainly which of the two possible outcomes will be obtained. In all, such experiments, that there is an element of chance, called probability which express the element of chance numerically. The occurrence or non-occurrence of things is studied in mathematics by the theory of probability. The theory of probability was introduced to give quantification to the possibility of certain outcome of the experiment in the face of uncertainty. Probability is used by governments, scientists, insurance company, etc to predict what is likely to happen in the future by studying what has already happened. In this unit you will learn the introductory concept of probability.

8.1. The concept of probability

Competency:

At the end of this sub-unit, students should:

- Describe the concepts of probability.

Definition 8.1:

An activity involving chance in which results are observed is called an experiment.

Definition 8.2:

Each observation of an experiment is called a trial and each result of the experiment is an outcome.

Note: The experiment in probability has the following properties.

- All the outcomes are known in advance.
- What specific outcome will result is not known in advance.
- The experiment can be repeated under same conditions.

Example 8.1:

Toss a coin once.

Experiment: Tossing a coin

Trial: one time

Outcome: Head (H), Tail (T)

Figure 8.1:

Definition 8.3:

The set of all possible outcomes of an experiment is called a sample space or possibility set of the experiment and is denoted by S.

Definition 8.4:

Any set of outcomes of the experiment is called an event. That is, event is a subset of the sample space. Events will be denoted by the capital letters A, B, C, D, E, and so on.

Example 8.2:

Consider the experiment "throwing a die once".

(A die is a cube with each of its six faces marked with a different number of dots from one to six)

Figure 8.2:

What are the possible outcomes?

a. Write the sample space. Figure 8.2
b. Write the event of a number 5 appearing on the upper face.
c. Write the event of getting "the number shown on the upper face is even".
d. Write the event of getting "a number different from three".
e. Write the event of getting "a prime number and an even number".

Solution:

a. The possible outcomes: 1, 2, 3, 4, 5, 6
b. S = {1, 2, 3, 4, 5, 6}
c. E = {5}
d. E = {2, 4, 6}
e. E = {1, 2, 4, 5, 6}
f. E = {2}

Exercise 8.1.

1. Discuss the following terms

a. Experiment b. Possibility set c. Event

2. Write a number 1 through 10 on a ten identical cards. Then select one card randomly from the cards.

a. What is the experiment? c. Write the sample space.
b. Write the event of getting an odd number.

Definition 8.5:

Events (two or more) of an experiment are said to be equally likely, if any one of them cannot be expected to occur in preference to the others.

Example 8.3:

a. Getting a 1, 2, 3 on the toss of a die and getting a 4, 5, 6 on the toss of a die are equally likely because the chance of occurrence of each event are equal.

b. A bag has 10 balls of different colors and sizes. If you pick up a ball randomly from the bag, the chance of occurrence of all the balls are not necessarily the same. Such outcomes are not equally likely outcomes.

Activity: 8.1.

1. Consider the experiment "throwing a fair die once". What can you say about the events
   a. The number on the upper face is a whole number.
   b. The number on the upper face is the number 7.
2. Identify the following events are certain or impossible events.
   a. A newly born baby will be a boy or a girl.
   b. Rolling a die and getting a number less than 7.
   c. A glass jar contains 15 red marbles and selecting a single white marble randomly from the jar.

Definition 8.6:

An event which is sure to occur at every performance of an experiment is called a certain event to the experiment.

Definition 8.7:

An event which cannot occur at any performance of the experiment is called an impossible event to the experiment.

Example 8.4:

a. A teacher chooses a girl from a class of 30 girls is a certain event. Because all the students in the class are girls, the teacher is certain to choose a girl.

b. A spinner has 4 equal sectors colored yellow, red, green and blue. It is impossible to land on purple after spinning the spinner.

Exercise 8.2.

1. A glass jar contains 3 red, 6 blue and 4 green chocolates. If a chocolate is chosen at random from the jar, then which of the following is an impossible event?

   a. Choosing a red chocolate.

   b. Choosing a yellow chocolate.

   c. Choosing a green chocolate.

2. A spinner has 6 equal sectors numbered 1 to 6. If you spin the spinner, then which of the following is a certain event?

   a. Landing on a number greater than 1.

   b. Landing on a number less than 7.

   c. Landing on a number between 6 and 9.

Figure 8.3:

3. Identify certain and impossible events

   a. You will live to be 300 years.

   b. Two lines intersect at one point.

   c. The earth revolves around the sun.

Definition 8.8:

The probability of an event is the measure of the chance that the event will occur as a result of the experiment.

The probability of an event E , denoted by P(E), is a number between 0 and 1, inclusive, that measures the likelihood of an event in the following way:

If $P(E_1) > P(E_2)$, then event E_1 is more likely to occur than event E_2

If $P(E_1) = P(E_2)$, then the events E_1 and E_2 are equally likely to occur.

If event E_1 is impossible, then $P(E_1) = 0$.

If event E_1 is certain, then $P(E_1) = 1$.

This can be shown on a probability scale, starting at 0(impossible) and ending at 1(certain)

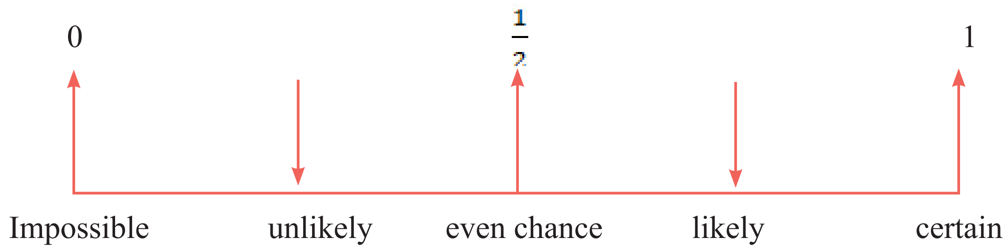


Figure 8.4:

Example 8.5:

Decide whether or not each of the statements below is reasonable.

a. The probability that you will go to bed before midnight tonight is 0.99.
b. The probability that your pocket money is doubled tomorrow is 0.01.

Solution:

a. This is a reasonable statement as it is very likely that you will go to bed before midnight, but not certain that you will.

b. This is a reasonable statement, as it is very unlikely that your pocket money will be doubled tomorrow, but not totally impossible.

Example 8.6:

On a probability scale, mark and estimate the probability that

a. It will rain on today in Hawass.
b. Ethiopia will win the next world cup.
c. Someone in your class has a birthday tomorrow.

Solution:

a. This will depend on the time of year and prevailing weather conditions.

During a dry season:

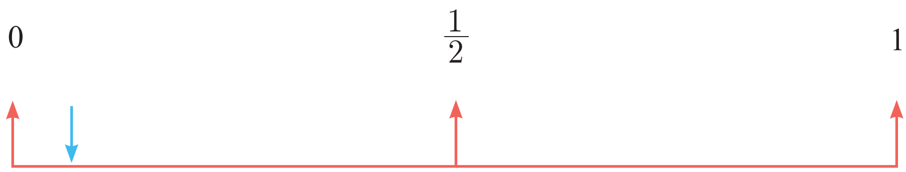


Figure 8.5:

During a summer season

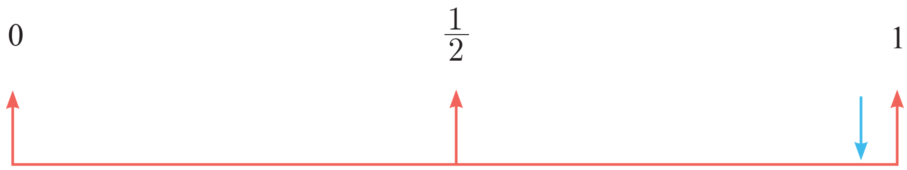


Figure 8.6:

b. Based on their recent performance, it is reasonable to say that Ethiopia are slightly more unlikely to win the world cup.

c.

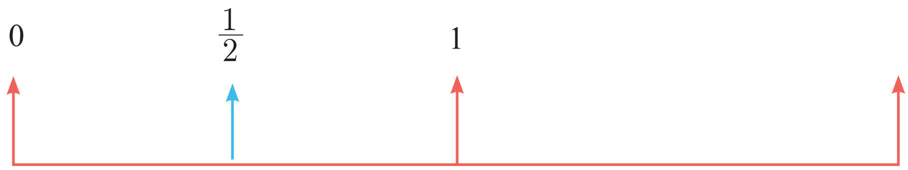


Figure 8.7:

d. The probability of this will be fairly small, as you can expect there to be about 2 or 3 birth days per month for students in a class of about 45 students.

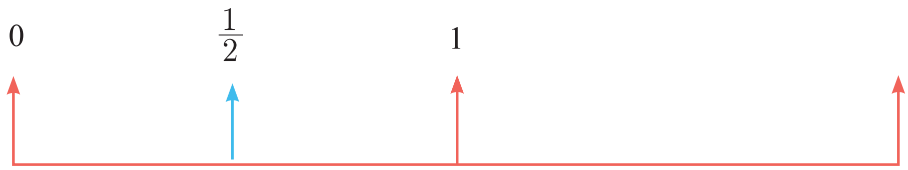


Figure 8.8:

Note:	probabilities are given on a scale of 0 to 1, as decimals or fractions; sometimes probabilities are expressed as percentages using a scale of 0% to 100% particularly on weather forecasts.

Exercise 8.3.

1. Give at least two examples of events that you think
   a. are impossible events.
   b. are certain events.
   c. are likely.
   d. are unlikely.
   e. have an even chance.
2. Draw a probability scale and mark on where you find the following events
   a. If you toss a coin, you will get a tail.
   b. It will rain at least one day on July in Addis Ababa.
   c. You will win a lottery.
   d. If you roll a die, you will get 6.
   e. You have already had a birthday this year.

8.3. Probability of Simple events

Competency:

At the end of this sub-unit, students should:

- Find the probability of simple events.

In this sub-unit we consider the probabilities of equally likely outcomes, experiment and associated sample space and event, and the different methods of finding the probability of an event.

What does a simple event mean? A simple event is an event containing exactly one outcome. For instance, in a toss of one coin, the occurrence of head is a simple event.

Historical Note

Mathematics was only one area of interest for Gerolamo Cardano.Gambling led Cardano in the 16th century to the study of probability, and he was the first writer of Games of Chance on the subject probability. It is the major starting point of the study of mathematical Probability.

Figure 8.9:

The following definition of probability is a theoretical approach of probability.

Definition 8.9

Let S be the possibility set of an experiment and each element of S be equally likely to occur. Then the probability of the event E occurring is given by

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{number of all possible outcomes}} = \frac{n(E)}{n(S)}$$

Example 8.7:

A box contains 5green balls and 3black balls. If one ball is drawn at random, what is the probability of getting a

a. Black ball b. Green ball

Solution:

Let event B = a black ball appears and event G = a green ball appears. There are 8 possible outcomes. Then

a. $P(B) = \frac{n(E)}{\ } = \frac{3}{\ }$

b. $P(G) = \frac{n(E)}{n(S)} = \frac{5}{8}$

Example 8.8:

A die is thrown once. What is the probability that the number appearing on the upper face will be

a. 6?

b. 7?

c. a number greater than 3

d. a number smaller than 9

Solution:

There are 6 possible outcomes. Hence, n(S) = 6

a. Only one of these outcomes is 6. Hence, n(E) = 1
Therefore, $P(E) = \frac{n(E)}{n(S)} = \frac{1}{6}$

b. 7 do not appear on the experiment. Hence, n(E) = 0
Therefore, $P(E) = \frac{n(E)}{n(S)} = 0$

c. 4, 5, 6 are the required elements. Hence, n(E) = 3
Therefore, $P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

d. All possible outcomes are smaller than 9. Hence n(E) = 6
Therefore, $P(E) = \frac{n(E)}{n(S)} = \frac{6}{6} = 1$

Example 8.9:

An integer is chosen at random out of the integers 1 to 50. What is the probability that it is

a. multiple 6

b. divisible by 11

c. greater than 38

Solution:

The total number of possible outcomes = 50

a. The favorable outcomes for the event "a multiple of 6" are 6, 12, 18, 24, 30, 36, 42, 48.
Hence, n(favorable outcomes E) = 8.
Therefore, $P(E) = \frac{n(E)}{n(S)} = \frac{8}{50} = \frac{4}{25}$

b. The favorable outcomes for the event "divisible by 11" are 11, 22, 33, and 44. Hence, n(favorable outcomes E) = 4
Therefore, $P(E) = \frac{n(E)}{n(S)} = \frac{4}{50} = \frac{2}{25}$

c. 39, 40, 41, 42, 43, 44, 45, 46 ,47, 48, 49, 50 are the required events.
Hence, n(E) = 12

Therefore, $P(E) = \frac{n(E)}{n(S)} = \frac{12}{50} = \frac{6}{25}$

Example 8.10:

A card is selected at random from a pack of 52 playing cards. What is the probability that it is:

a. A red card
b. A queen
c. A red "Ace"
d. an even number
e. the 7 of hearts

Solution:

A standard deck of playing cards consists of 52 cards in each of the four suits of Spades, Hearts, Diamonds, and clubs. Each suit contains 13 cards:

Ace, 2, 3, ….10, Jack, Queen, and King.

As each card is equally likely to be drawn from the pack,

a. There are 26 red cards in the pack, so:
   $P(\text{red}) = \frac{26}{52} = \frac{1}{2}$

b. There are 4 Queens in the pack, so:
   $P(\text{queens}) = \frac{4}{52} = \frac{1}{13}$

   There are 2 red Aces in the pack, so:

   $P(\text{red Ace}) = \frac{2}{52} = \frac{1}{26}$

Figure 8.10:

c. There are 20 cards that have even numbers in the pack, so:
   $P(\text{even number}) = \frac{20}{52} = \frac{5}{13}$

d. There is only one 7 of hearts in the pack, so:
   $P(7 \text{ of hearts}) = \frac{1}{52}$

Example 8.11:

Toss a coin twice (or toss two coins once at a time).

a. Find the possible out comes
b. Find the probability of getting exactly two heads.
c. Find the probability of getting at least one tail.

Solution:

a. The possible outcomes are: HH, HT, TH, TT.
Therefore, the total number of possible outcomes, n(S) = 4

b. The favorable outcome for the event "exactly two heads" is HH.
Hence, the total number favorable outcomes = 1
Therefore, P(exactly two heads) = $\frac{1}{4}$

c. The favorable outcomes for the event "at least one tail" are HT, TH, TT.
Hence, the total number favorable outcomes = 3
Therefore, P(at least one tail) = $\frac{3}{4}$

Example 8.12:

Determine the probability that the sum 7 appear in a single toss of a pair of fair dice?

Solution:

Each of the six faces of one die can be associated with each of the six faces of the other die, so that the total number of cases that can arise are all equally likely is $6 \times 6 = 36$. These can be denoted by (1,1), (2,1),….,(6,6)

There are six ways of obtaining the sum 7, denoted by (1,6), (2,5), (3,4), (5,2), (4,3), and (6,1).

Thus p(sum 7) = $\frac{6}{36} = \frac{1}{6}$

Example 8.13:

If a family has three children, find the probability that two of the three children are girls.

Solution:

The sample space for the gender of the children for a family that has three children has eight outcomes, that is, BBB, BBG, BGB, GBB, GGG, GGB,GBG and BGG. Since there are three ways to have two girls, namely: GGB, GBG, and BGG.

Therefore, P(two girls) = $\frac{3}{8}$.

Exercise 8.4.

1. A spinner is numbered as shown below in the diagram. Each score is equally likely to occur. What is the probability of scoring

   a. 1 c. 3
   b. 2 d. 4
   e. a number less than 5?

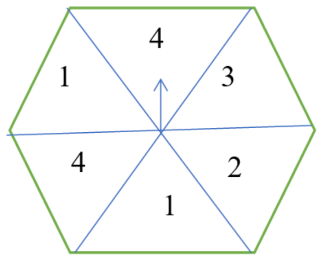


Figure 8.11:

2. A letter is chosen at random from the word" ETHIOPIA". Find the probability that it will be:

   a. the letter I?
   b. a vowel?
   c. the letter B?

3. Describe something that has a probability of zero? One?
4. The probability that a train is late is 0.25. Which of the following is the most reasonable?

   a. The train is unlikely to be late.
   b. The train is likely to be late.
   c. The train is impossible to come.

In the above discussion, you saw how to determine probabilities of events of equally likely outcomes using theoretical approach. The following group work introduces you the other approach of probability.

Group work 8.2

Perform the following experiment repeatedly and observe your result.

a. Toss a coin 10 times, 20 times, and 30 times
b. Record your results on landing of coin on Tails.

Number of tosses	Number of Tails
10	
20	
30	

- For each row in the table, what proportion of the number of tosses landed as tails? How do your answers compare with P(Tail) = $\frac{1}{2}$?
- If you toss a coin 480 times, how many times would you expect to obtain a head?

From the above experiment, that is, based on the frequency of an outcome when an experiment is repeated a large number of times under the same conditions, the proportion of times that the outcome is contained in event E approaches some value as the number of repetitions increase and we derive the formula of the probability of a simple event as:

Definition 8.10:

The probability of an event is the ratio of the number of successful outcomes in the event to the total number of possible outcomes in the experiment.

Thus, P($EventEvent$) = $\frac{Number\ of\ times\ the\ event\ occur}{Total\ number\ of\ observations}$

[experimental approach of probability]

Example 8.14:

If record show that getting 120 tails out of 10,000 trial of throwing a coin, then the probability of getting another tail is given by

$$P(EventEvent) = \frac{Number\ of\ times\ the\ event\ occur}{Total\ number\ of\ observations}$$

$$= \frac{120 \quad 120}{1000010000} = 0.012$$

Example 8.15:

You roll a fair dice (Plural form die) 240 times. How many times would you expect to obtain:

a. 4?

b. an even score

c. a score greater than 4

Solution:

a. P(4) = $\frac{1}{6}$ (Theoretical probability)

Expected number of 4's = $\frac{1}{6} \times 240 = 40$

b. P(Even score) = $\frac{1}{2}$

Expected number of even scores = $\frac{1}{2} \times 240 = 120$

c. P(score greater than 4) = $\frac{2}{6} = \frac{1}{3}$

Expected number of scores greater than 4 = $\frac{1}{3} \times 240 = 80$

8.4. Applications on Business, Climate, Road Transport,

8.4.1. Accidents and Drug Effects

Competency:

At the end of this sub-unit, students should:

- Solve the probability of real-life problems.

The preceding sections contained a discussion of the meaning and different ways of viewing probability. This section focuses on some real –life applications of probability.

Example 8.16:

At a car park there are 52 vehicles, 26 of which are cars, 8 are vans and the remaining Lorries. If every vehicle is equally likely to leave, find the probability of

a. A van leaving first.

b. A lorry leaving first.

Solution:

a. Let E be the event of van leaving first. Then n(E) = 8

$$P(\text{van leaving first}) = P(E) = \frac{n(E)}{n(S)} = \frac{8}{52} = \frac{2}{13}$$

b. Let A be the vent of a lorry leaving first. Then

n(A) = 52 – 26 - 8 = 18

$$P(\text{lorry leaving first}) = P(A) = \frac{n(E)}{n(S)} = \frac{18}{52} = \frac{9}{26}$$

Example 8.17:

A factory has 2 machines A and B producing 300 and 720 bulbs per day respectively. A produces 1% defective and B produces 1.5% defective. A bulb is chosen at random at the end of a day and found defective. What is the probability that

a. Machine A produces a defective bulb
b. Machine B produces a defective bulb.

Solution:

a. Number of defective bulbs produced by machine A = 1% × 300=3

Therefore, the probability of the chosen bulb is defective is

$$P(\text{defective bulb}) = \frac{3}{300} = 0.01$$

b. Number of defective bulbs produced by machine B = 1.5% × 720 = 108

Therefore, the probability of the chosen bulb is defective is

$$P(\text{defective bulb}) = \frac{108}{720} = 0.15$$

Example 8.18:

Suppose that a researcher for the road transport office asked 350 people who plan to travel over the Epiphany holiday how they will get to Gondar from Addis Ababa. The result can be categorized as: drive-85, fly-115 and bus-150. In the travel survey, just described find the probability that a person will travel by bus over the Epiphany holiday?

Solution:

Total number of traveller = 350

Number of traveller by using bus = 150 = n(E)

$$P(Event) = \frac{Number\ of\ times\ the\ event\ occur}{Total\ number\ of\ observations}$$

$$P(E) = \frac{150}{350}$$

$$= \frac{3}{7}$$

Example 8.19:

The record of a weather station shows that out of the past 250 consecutive days, its weather forecasts were correct 205 times. What is the probability that

a. On a given day it was correct?

b. It was not correct on a given day?

Solution:

a. $P(\text{correct}) = \frac{number\ of\ days\ when\ the\ forecast\ was\ correct}{total\ number\ of\ days\ for\ which\ the\ records\ is\ available}$

$$= \frac{205}{250}$$

$$= 0.82 \quad = \ 82\%$$

Therefore, 82% of the next forecast will be correct.

b. $P(\text{incorrect}) = \frac{number\ of\ days\ when\ the\ forecast\ was\ not\ correct}{total\ number\ of\ days\ for\ which\ the\ records\ is\ available}$

$$= \frac{250 - 205}{250}$$

$$= \frac{45}{250}$$

$$= 0.18 \ = \ 18\%$$

Therefore, 18% of the next forecast will be incorrect.

Example 8.21:

A company receives regular deliveries of raw materials from a supplier. The supplies do not always arrive on time. Over the last 120 delivery days, supplies have been late on 15 occasions. What is the probability that the supplies will be on time on the next delivery?

Solution:

Number of days arrive on time = 120 – 15 = 105.

P(arrive on time) = $\frac{105}{120}$ = 0.875 = 87.5%

Therefore, the on time delivery the supply will be 87.5%.

Example 8.22:

According to Addis Ababa Annual Road Safety Report; The following table presents summary findings on fatally injured victims from road traffic crashes in 2011 E.C in Addis Ababa.

Deaths	Fatal crashes	Injury crashes	Property damage	Total
458	1875	1115	26000	29448

Find the probability of

a. Being killed in a road traffic crash.
b. Being injured in a road traffic crash.

Solution:

a. P(deaths) = $\frac{458}{29448} = 0.016 = 1.6\%$

b. P(injured) = $\frac{1875 + 1115}{29448} = \frac{2990}{29448} = 0.102 = 1.02\%$

In Addis Ababa, 1.6% of victims died and 1.o2% of victims injured due to road traffic crashes.

Example 8.23:

Are we likely to die in an accident? For instance,

Per 100,000 populations in Addis Ababa the death rate in 2011 was 13.3. Find the probability of being to die in a car accident?

Solution:

The chance of being killed in a crash = $\frac{13.3}{100000} = 0.000133 = 0.0133\%$

Thus, a person has a 0.0133% chance to die in a car accident.

UNIT SUMMARY

1. A process of an observation is often called an experiment.
2. The possibility set for an experiment is the set of all possible outcomes of the experiment. It also known as the sample space.
3. An event is a favorable outcome from the possibility set.
4. If S is the possibility set of an experiment and each element of S is equally likely, then the probability of an event E occurring, denoted by P(E), is defined as

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{number of all possible outcomes}} = \frac{n(E)}{n(S)}$$

5. Probabilities can be expressed as fractions, decimals, or where appropriate percentages.
6. An event which is certain to happen has a probability of 1.
7. An event which cannot happen is impossible event and has a probability of 0.
8. The probability of an event is the ratio of the number of successful outcomes in the event to the total number of possible outcomes in the experiment.

   Thus, $P(Event) = \frac{\text{Number of times the event occur}}{\text{Total number of observations}}$

   [experimental approach of probability]
9. The probability of any event E is a number between and including 0 and 1. This is denoted by $0 \leq P(E) \leq 1$. Therefore, probabilities cannot be negative or greater than 1.

REVIEW EXERCISE

1. Write true if the statement is correct and false if it is not.
   a. The probability of an event that is sure to occur is $\frac{1}{2}$.
   b. If three coins are thrown at a time, then n(S) = 8
   c. If the set of all possible outcomes is an event, the probability of an event is 1.

d. The range of the values of the probability of an event is between 0 and 1.

2. Answer the following questions.

   a. What is a probability experiment?
   b. Define sample space.
   c. What is the difference between an outcome and an event?
   d. What are equally likely events?
   e. If an event cannot happen, what value is assigned to its probability?

3. Which of the following statements describe a certain event?

   a. Roll a die once and the event of getting a number less than 7.
   b. Toss a coin twice and the event of getting two tails.
   c. Out of a family of 4girls the event of selecting two boys.

4. Suppose you write the days of the week on identical pieces of paper. Mix them and select one at a time. What is the probability that the day you select will have

   a. The letter *r* in it.
   b. The letter *d* in it.
   c. The letter *b* in it.

5. A number is chosen out of the numbers 1 through 30. What is the probability that it is:

   a. A multiple of 6?
   b. A perfect square?

6. A coin has two sides, heads and tails

   a. Surafel is going to toss a coin. What is the probability that Surafel will get heads? Write your answer as a fraction and decimal.
   b. Meseret is going to toss 2 coins. Copy and complete the following table to show the different results she could get.

First toss	**Second toss**
Heads	Heads

   c. What is the probability that she will get tails with both her coins?

7. If a die is rolled one time, find these probabilities
   a. Of getting a 4
   b. Of getting an even number
   c. Of getting a number greater than 4
   d. Of getting a number less than 7
8. Suppose a pair of dice is thrown.
   a. Set up a sample space.
   b. What is the probability that
      i. The sum of the scores is 7?
      ii. Both numbers are the same?
      iii. The first number is odd and the second number is even.
      iv. A sum is greater than 9
9. There are 18 tickets marked with numbers 1 to 18. What is the probability of selecting a ticket having the following property?
   a. Even number
   b. Number divisible by 3
   c. Prime number
   d. Number divisible by 4
10. There are 20 pens in a box, 13 of the pens are blue. 7 of the pens are black. If one pen is chosen random, what is the probability of getting
   a. Blue pen?
   b. Red pen?
   c. Both color?
11. A spinner is marked with the letters A, B, C, and D, so that each letter is equally likely to be obtained. The spinner is spun twice.
   a. List the 16 possible outcomes.
   b. What is the probability that
   v. A is obtained twice
   vi. A is obtained at least once.
   vii. both letters are the same.
   viii. the letter B is not obtained at all
12. In a class there are 12 boys and 14 girls. The teacher calls on a student at random to read a theorem stated on a text book. What is the probability that the first person called a girl?
13. There are seven cups of coffee on a table. Three of them contain sugar. What is your chance of choosing a cup with sugar in it?

The Function $y = x^2$ **$1.00 \leq x \leq \leq x \leq 5.69$**

X	0	1	2	3	4	5	6	7	8	9
1.0	1.000	1.020	1.040	1.061	1.082	1.102	1.124	1.145	1.166	1.188
1.1	1.210	1.232	1.254	1.277	1.277	1.322	1.346	1.369	1.392	1.416
1.2	1.440	1.464	1.488	1.513	1.513	1.562	1.588	1.613	1.638	1.644
1.3	1.690	1.716	1.742	1.769	1.769	1.822	1850	1.877	1.904	1.932
1.4	1.960	1.988	2.016	2.045	2.045	2.102	2.132	2.161	2.190	2.220
1.5	2.250	2.280	2316	2.341	2.341	2.102	2.434	2.465	2.496	2.528
1.6	2.560	2.592	2.310	2.657	2.657	2.722	2.756	2.789	2.822	2.856
1.7	2.890	2.924	2.624	2.993	2.993	3.062	3.098	3.133	3.168	3.204
1.8	3.240	3.276	2.958	3.349	3.349	3.422	3.460	3.497	3.534	3.572
1.9	3.610	3.648	3.686	3.725	3.764	3.802	3.842	3.881	3.920	3.960
2.0	4.000	4.040	4.080	4.121	4.162	4.202	4.244	4.285	4.326	4.368
2.1	4.410	4.452	4.494	4.537	4.580	4.622	4.666	4.709	4.752	4.796
2.2	4.840	4.884	4.928	4.973	5.018	5.062	5.108	5.153	5.198	5.244
2.3	5.290	5.336	5.382	5.429	5.476	5.522	5.570	5.617	5.664	5.712
2.4	5.760	5.808	5.856	5.905	5.954	6.002	6.052	6.101	6.150	6.200
2.5	6.250	6.300	6.350	6.401	6.452	6.502	6.554	6.605	6.656	6.708
2.6	6.760	6.812	6.864	6.917	6.970	7.022	7.076	7.129	7.182	7.236
2.7	7.290	7.344	7.398	7.453	7.508	7.562	7.618	7.673	7.728	7.784
2.8	7.840	7.896	7.952	8.009	8.066	8.122	8.180	8.237	8.294	8.352
2.9	8.410	8.468	8.526	8.585	8.644	8.702	8.762	8.821	8.880	8.940
3.0	9.000	9.060	9.120	9.181	9.242	9.302	9.364	9.425	9.486	9.548
3.1	9.610	9.672	9.734	9.797	9.860	9.922	9.986	10.05	10.11	10.18
3.2	10.24	10.30	10.37	10.43	10.50	10.56	10.63	10.69	10.76	11.82
3.3	10.89	10.96	11.02	11.09	11.16	11.22	11.29	11.36	11.42	11.49
3.4	11.56	11.63	11.70	11.76	11.83	11.90	11.97	12.04	12.11	12.18
3.5	14.25	12.32	12.39	12.46	12.53	12.60	12.67	12.74	12.82	12.89
3.6	12.96	13.03	13.10	13.18	13.25	13.32	13.40	13.47	13.54	13.62
3.7	13.69	13.76	13.84	13.91	13.99	14.06	14.14	14.21	14.29	14.36
3.8	14.44	14.52	14.59	14.67	14.75	14.82	14.90	14.98	15.08	15.13
3.9	15.21	15.29	15.37	15.44	15.52	15.60	15.68	15.76	15.84	15.92
4.0	16.00	16.08	16.16	16.24	16.32	16.40	16.48	16.56	16.65	16.73
4.1	16.81	16.89	16.97	17.06	17.14	17.22	17.31	17.39	17.47	17.56
4.2	17.64	17.72	17.81	17.89	17.98	18.06	18.15	18.23	18.32	18.40
4.3	18.49	18.58	18.66	18.75	18.84	18.92	19.01	19.10	19.18	19.27
4.4	19.96	19.45	19.54	19.62	19.71	19.80	19.89	19.98	20.98	20.16
4.5	20.25	20.34	20.43	20.52	20.61	20.70	20.79	20.88	21.90	21.07
4.6	21.16	21.25	21.34	21.44	21.53	21.62	21.72	21.81	22.85	22.00
4.7	22.09	22.18	22.28	22.37	22.47	22.56	22.66	22.75	23.81	22.94
4.8	23.04	23.14	23.33	23.33	23.43	23.52	23.62	23.72	24.80	23.91
4.9	24.01	24.11	24.24	24.30	24.40	24.50	24.60	24.70	25.81	24.90
5.0	25.00	25.10	25.20	25.30	25.40	25.50	25.60	25.70	26.83	25.91
5.1	26.01	26.11	26.21	26.32	26.42	26.52	26.63	26.73	27.88	26.94
5.2	27.04	27.14	27.25	27.35	27.46	27.56	27.67	27.77	28.94	27.98
5.3	28.09	28.20	28.30	28.41	28.52	28.62	28.73	28.84	28.94	29.05
5.4	29.16	29.27	29.38	29.48	29.59	29.70	29.81	29.92	30.03	30.14
5.5	30.25	30.36	30.47	30.58	30.69	30.80	30.91	31.02	31.14	31.25
5.6	31.36	31.47	31.58	31.70	31.81	31.92	32.04	32.15	32.26	32.38
5.7	32.46	32.60	32.72	32.83	32.95	33.06	33.18	33.29	33.41	33.52
5.8	33.64	33.76	33.87	33.99	34.11	34.22	34.34	34.46	34.57	34.69
5.9	34.81	34.93	35.05	35.16	35.28	35.40	35.52	35.64	35.76	35.88

The Function $y = x^2$ **$5.70 \leq x \leq 9.99 \leq x \leq 9.99$**

6.0	36.00	36.12	36.24	36.36	36.48	36.60	36.72	36.84	36.97	37.09
6.1	37.21	37.33	37.45	37.58	37.70	37.82	37.95	38.07	38.19	38.32
6.2	38.44	38.56	38.69	38.81	38.94	39.06	39.19	39.31	39.44	39.56
6.3	39.69	39.82	39.94	40.07	40.20	40.32	40.45	40.58	40.70	40.83
6.4	40.96	41.09	41.22	41.34	41.47	41.60	41.73	41.86	41.99	42.12
6.5	42.25	42.38	42.51	42.64	42.77	42.90	43.03	43.16	43.30	43.43
6.6	43.56	43.69	43.82	43.96	44.09	44.22	44.36	44.49	44.62	44.47
6.7	44.89	45.02	45.16	45.29	45.43	45.56	45.70	45.63	45.63	46.10
6.8	46.24	46.38	46.51	46.65	46.79	46.92	47.06	47.20	47.20	47.47
6.9	47.61	47.75	47.89	48.02	48.16	48.30	48.44	48.58	48.58	48.86
7.0	49.00	49.14	49.28	49.42	49.56	49.70	49.84	49.98	49.98	50.27
7.1	50.41	50.55	50.69	50.84	50.98	51.12	51.27	51.41	54.41	51.70
7.2	51.84	51.98	52.13	52.27	52.42	52.56	52.71	52.85	52.85	53.14
7.3	53.29	53.44	53.58	53.73	53.88	54.02	54.17	54.32	54.32	54.61
7.4	54.76	54.91	55.06	55.20	55.35	55.50	55.65	55.80	55.80	56.10
7.5	56.25	56.40	56.55	56.70	56.85	57.00	57.15	57.30	57.30	57.61
7.6	57.76	57.91	58.06	58.22	58.37	58.52	58.68	58.83	58.83	59.14
7.7	59.29	59.44	59.60	59.75	59.91	60.06	60.22	60.37	60.37	60.68
7.8	60.84	61.00	61.15	61.31	61.47	61.62	61.78	61.94	61.94	62.25
7.9	62.41	62.57	62.73	62.88	63.04	63.20	63.36	63.52	63.52	63.84
8.0	64.00	64.16	64.32	64.48	64.64	64.80	64.96	65.12	65.29	65.45
8.1	65.61	65.77	65.93	66.10	66.26	66.42	66.59	66.75	66.91	67.08
8.2	67.24	67.40	67.57	67.73	67.90	68.06	68.23	68.39	68.56	68.72
8.3	68.29	69.06	69.22	69.39	69.56	69.72	69.89	70.06	70.22	70.39
8.4	70.56	70.73	70.90	71.06	71.23	71.40	71.57	71.74	71.91	72.01
8.5	72.25	72.42	72.59	72.76	72.93	73.10	73.27	73.44	73.62	73.79
8.6	73.96	74.13	74.30	74.48	74.65	74.82	75.00	75.17	75.34	75.52
8.7	75.69	75.86	76.04	76.21	76.39	76.56	76.74	76.91	76.09	77.26
8.8	77.44	77.62	77.79	77.97	78.15	78.32	78.50	78.68	78.85	79.03
8.9	79.21	79.39	79.57	79.74	79.92	80.10	80.28	80.46	80.46	80.82
9.0	81.00	81.18	84.36	81.54	81.72	81.90	82.08	82.26	82.45	82.63
9.1	82.81	82.99	83.17	83.36	83.54	83.72	83.91	84.09	86.12	84.46
9.2	84.64	84.82	85.01	85.19	85.38	85.56	85.75	84.93	87.96	86.30
9.3	86.49	86.68	86.86	87.05	87.24	87.42	87.61	87.80	87.96	88.17
9.4	88.36	88.55	88.74	88.92	89.11	89.30	89.49	89.68	89.87	90.06
9.5	90.25	90.44	90.63	90.82	91.01	91.20	91.39	91.58	91.78	91.97
9.6	92.16	92.35	92.54	92.74	92.93	93.12	93.32	93.51	93.70	93.90
9.7	94.09	94.28	94.67	94.67	94.87	95.06	95.26	95.45	95.65	95.84
9.8	96.04	96.24	96.43	96.63	96.83	97.02	97.22	97.42	97.61	97.81
9.9	98.01	98.21	98.41	98.60	98.80	99.00	99.20	99.40	99.60	99.80

$(7.95)^2 = 63.20$, $(0.795)^2 = 0.6320$, $(79.5)^2 = 6320$, $\sqrt{23.33}\sqrt{23.33} = 4.83$, $\sqrt{2333}\sqrt{2333} = 48.3$, $\sqrt{0.2333}\sqrt{0.2333} = 0.483$

If you move the comma in x one digit to the right (left), then the comma in x^2 must be two digits to the right (left)

MATHEMATICS

STUDENT'S BOOK

GRADE 8

SNNPR Education Bureau